






# An Improved Epsilon Method with M2M for Solving Imbalanced CMOPs with Simultaneous Convergence-Hard and Diversity-Hard Constraints

Zhun Fan<sup>1,2,3,4</sup> , Zhi Yang<sup>1</sup>, Yajuan Tang<sup>1</sup> , Wenji Li<sup>1</sup> , Biao Xu<sup>1</sup>, Zhaojun Wang<sup>1</sup>, Fuzan Sun<sup>1</sup>, Zhoubin Long<sup>1</sup>, and Guijie Zhu<sup>1</sup>

<sup>1</sup> Department of Electronic and Information Engineering, Shantou University, Shantou 515063, Guangdong, China

zfan@stu.edu.cn

<sup>2</sup> Key Lab of Digital Signal and Image Processing of Guangdong Province, Shantou University, Shantou 515063, Guangdong, China

<sup>3</sup> Key Laboratory of Intelligent Manufacturing Technology (Shantou University), Ministry of Education, Shantou 515063, Guangdong, China

<sup>4</sup> State Key Lab of Digital Manufacturing Equipment and Technology, Huazhong University of Science and Technology, Wuhan 430000, China

**Abstract.** When tackling imbalanced constrained multi-objective optimization problems (CMOPs) with simultaneous convergence-hard and diversity-hard constraints, a critical issue is to balance the diversity and convergence of populations. To address this issue, this paper proposes a hybrid algorithm which combines an improved epsilon constraint-handling method (IEpsilon) with a multi-objective to multi-objective (M2M) decomposition approach, namely M2M-IEpsilon. The M2M decomposition mechanism in M2M-IEpsilon has the capability to deal with imbalanced objectives. The IEpsilon constraint-handling method can prevent populations falling into large infeasible regions, thus improves the convergence performance of the proposed algorithm. To verify the performance of the proposed M2M-IEpsilon, a series of imbalanced CMOPs with simultaneous convergence-hard and diversity-hard constraints, namely ICD-CMOPs, is designed by using the DAS-CMOPs framework. Six state-of-the-art constrained multi-objective evolutionary algorithms (CMOEAs), including CM2M, CM2M2, NSGA-II-CDP, MOEA/D-CDP, MOEA/D-IEpsilon and PPS-MOEA/D, are employed to compare with M2M-IEpsilon on the ICD-CMOPs. Through the analysis of experimental results, the proposed M2M-IEpsilon is superior to the other six algorithms in solving ICD-CMOPs, which illustrates the superiority of the proposed M2M-IEpsilon in dealing with ICD-CMOPs with simultaneous convergence-hard and diversity-hard constraints.

**Keywords:** CMOEAs · M2M decomposition · IEpsilon · Imbalanced CMOPs

# 1 Introduction

Many optimization problems usually have conflicting objectives with a set of constraints [6, 13, 14]. Generally, a constrained multi-objective optimization problem (CMOP) is described as follows [8]:

$$\begin{cases} \text{minimize} & \mathbf{F}(\mathbf{x}) = (f_1(\mathbf{x}), \dots, f_m(\mathbf{x}))^T \\ \text{subject to} & g_i(\mathbf{x}) \geq 0, i = 1, \dots, q \\ & h_j(\mathbf{x}) = 0, j = 1, \dots, p \\ & \mathbf{x} \in \mathbf{R}^n \end{cases} \quad (1)$$

where  $\mathbf{F}(\mathbf{x}) = (f_1(\mathbf{x}), \dots, f_m(\mathbf{x}))^T$  represents an  $m$ -dimensional objective function.  $\mathbf{x} \in \mathbf{R}^n$  denotes an  $n$ -dimensional decision variable.  $g_i(\mathbf{x}) \geq 0$  and  $h_j(\mathbf{x}) = 0$  represent inequality and equality constraints, respectively.  $q$  and  $p$  are the number of inequality and equality constraints, respectively.

For any two feasible solutions  $x_p$  and  $x_q$ , we say that  $x_p$  dominates  $x_q$  ( $x_p \leq x_q$ ) when the following two conditions are met:

1.  $\forall i \in \{1, \dots, m\}, f_i(x_p) \leq f_i(x_q)$
2.  $\exists j \in \{1, \dots, m\}, f_j(x_p) < f_j(x_q)$

Some real-world optimization problems may have imbalanced objectives and a series of constraints. Therefore, CMOPs with imbalanced objective functions deserve to be solved. To handle with the imbalanced CMOPs, we propose a hybrid CMOEAs, named M2M-IEpsilon, which has adopted two mechanisms. Firstly, to enhance the diversity of the population, an M2M decomposition method [10] is used to handle with imbalanced objectives. Secondly, an improved epsilon constraint-handling method (IEpsilon) [3] is employed to improve the convergence performance of the method. In M2M-IEpsilon, these two mechanisms are combined to solve imbalanced CMOPs [9] with simultaneous diversity-hard and convergence-hard constraints. To verify the effectiveness of the M2M-IEpsilon, a series of imbalanced CMOPs with diversity-hard and convergence-hard constraints is designed by using DAS-CMOPs framework [5], which is named as ICD-CMOPs. The contributions of this paper are listed below:

1. The proposed M2M-IEpsilon decomposes a population into many sub-populations, with each sub-population corresponding to a CMOP, which can assist to solve imbalanced CMOPs.
2. M2M-IEpsilon can dynamically adjust the relaxation of constraint violation according to the proportion of feasible solutions in the current population, which maintains a good balance of searching between the feasible and infeasible regions, and has the ability to get across large infeasible regions.
3. A set of CMOPs, namely ICD-CMOPs, is constructed to assess the diversity and convergence performance of CMOEAs.

The remainder of this paper is organized as follows. Section 2 mainly introduce the M2M decomposition approach and the IEpsilon method. Section 3

describes the proposed M2M-IEpsilon. Section 4 shows the experimental results of M2M-IEpsilon and six other CMOEAs, including CM2M [11], CM2M2 [12], NSGA-II-CDP [2], MOEA/D-CDP [7], MOEA/D-IEpsilon [3] and PPS-MOEA/D [4], on ICD-CMOPs. Section 5 make a summary of this paper.

## 2 Background

### 2.1 The Framework of M2M

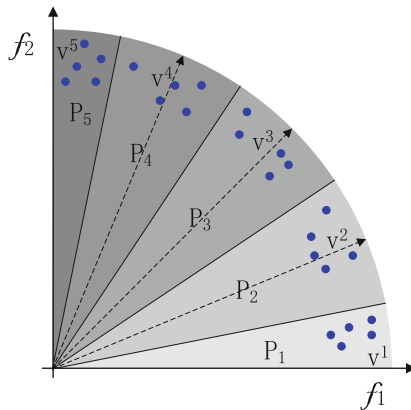
In M2M [10], the decomposition is carried out in the objective space. Specifically,  $J$  unit vectors  $v^1, \dots, v^j$  in  $\mathbf{R}_+^m$  are selected. The  $\mathbf{R}_+^m$  is split into  $J$  sub-regions  $P_1, \dots, P_J$ . A sub-region  $P_j$  is defined as follows:

$$P_j = \{u \in \mathbf{R}_+^m \mid \langle u, v^j \rangle \leq \langle u, v^i \rangle \text{ for any } i = 1, \dots, J\} \tag{2}$$

where  $\langle u, v^i \rangle$  is the acute angle between the objective vector  $u$  and the  $i$ th direction vector  $v^i$ . To search for optimal solutions in each sub-region, the population of M2M is separated into  $J$  sub-populations. The Eq. (3) shows the definition of a subproblem  $j$ . In addition, each sub-population adopts two rules in the Ref. [10] to select solutions.

$$\begin{cases} \text{minimize} & \mathbf{F}(x) = (f_1(x), \dots, f_m(x)) \\ \text{subject to} & x \in \prod_{i=1}^n [a_i, b_i] \\ & F(x) \in P_j \end{cases} \tag{3}$$

Figure 1 shows a decomposition example by using M2M. The objective space  $\mathbf{R}_+^m$  is separated into five sub-regions  $P_1, \dots, P_5$ . Each sub-region has  $S$  individuals, and  $v_1, \dots, v_5$  are five evenly distributed direction vectors.



**Fig. 1.** The population decomposition approach by using M2M.

## 2.2 The Improved Epsilon Constraint-Handling Method

In paper [3], an improved  $\varepsilon$  constraint-handling method is proposed, namely IEpsilon, which can dynamically adjust the relaxation of constraint violations based on the ratio of feasible solutions in each generation of the population. The detail description of the setting of  $\varepsilon(k)$  can be found in Ref. [3].

$$\varepsilon(k) = \begin{cases} \text{rule 1: } \phi(x^\theta), & \text{if } k = 0 \\ \text{rule 2: } (1 - \tau)\varepsilon(k - 1), & \text{if } r_k < \alpha \text{ and } k < T_c \\ \text{rule 3: } (1 + \tau)\phi_{max}, & \text{if } r_k \geq \alpha \text{ and } k < T_c \\ \text{rule 4: } 0, & \text{if } k \geq 0 \end{cases} \quad (4)$$

## 3 Embedding IEpsilon in M2M

In the M2M framework, the IEpsilon is embedded to solve constraints. The description of M2M-IEpsilon is presented in Algorithm 1. In line 2, the working population is decomposed into  $J$  sub-populations which are associated with evenly distributed direction vectors  $v^1, \dots, v^j$ , and each sub-population contains  $S$  individuals. Line 3 introduces several parameters in M2M-IEpsilon, including  $\varepsilon(0)$ ,  $r_k$ , and  $\phi_{max}$ . Lines 5–14 show the process of generate new offspring by using genetic operators. In lines 15–17,  $\varepsilon(k)$  is set according to Eq. (4). Lines 18–31 describe the updating process of each sup-population. Specially, if the number of solutions in sup-population  $P_j$  is fewer than  $S$ , then randomly select  $|S - P_j|$  solutions add to  $P_j$ . In lines 24–30, if the number of solutions in sup-population  $P_j$  is more than  $S$ , we apply two strategies to select  $S$  solutions. If the number of solutions whose constraint violations are smaller than  $\varepsilon(k)$ , denoted as  $P_{j_{fea}}$ , is equal to or greater than  $S$ , we select  $S$  solutions from  $P_{j_{fea}}$  by using NSGA-II [2]. Otherwise, we first select all the solutions in  $P_{j_{fea}}$  to the next generation. Then, we sort all solutions in  $P_{j_{inf}}$  according to their constraint violations in an ascending way, and select the first  $S - |P_{j_{fea}}|$  solutions to the next generation.

## 4 The Analysis of Experimental Study

### 4.1 Parameter Settings of Seven CMOEAs

The six state-of-the-art CMOEAs, including CM2M [11], CM2M2 [12], NSGA-II-CDP [2], MOEA/D-CDP [7], MOEA/D-IEpsilon [3], PPS-MOEA/D [4], and the proposed M2M-IEpsilon, are carried out on ICD-CMOP1-7. The detailed parameter settings are presented as follows:

1. Population size:  $N = 300$ .
2. Mutation and crossover probability:  $P_m = 1/n$ ,  $CR = 1.0$ .
3. Termination condition: all algorithm independently runs for 30 times on ICD-CMOP1-7, then stop when reach 300,000 function evaluations.

**Algorithm 1:** The Framework of the Proposed Algorithm (M2M-IEpsilon)**Input:**

$J$ : the number of the subproblems;  
 $J$  unit direction vectors:  $v^1, \dots, v^J$ ;  
 $S$ : the number of sub-population;  
 $Q$ : a group of individual solutions;  
 $T_{max}$ : the maximum generation;  
 $T_c$ : the control generation for  $\varepsilon(k)$ ;  
 $\alpha$ : the maximum overall constraint violation;  
parameters  $\tau$  and  $\theta$  in Eq.(4)

**Output:** a set of feasible non-dominated solutions.

```

1 Initialization:
2 Decompose the working population into  $J$  sub-populations ( $P_1, \dots, P_J$ ), each of
  them consists of  $S$  individuals by using Eq.(3)
3 Initialize  $\varepsilon(0)$ ,  $r_k$  and  $\phi_{max}$  according to Eq.(4)
4 while  $gen \leq T_{max}$  do
5   for  $j \leftarrow 1$  to  $J$  do
6     foreach  $x \in P_j$  do
7        $y$  be selected from  $P_j$ ;
8       A new solutions  $z$  be generated by applying genetic operators on  $x$ 
        and  $y$ ;
9       Compute  $F(z)$ ;
10       $R := R \cup \{z\}$ ;
11    end
12     $Q := R \cup (\cup_{j=1}^J P_j)$ ;
13    Use  $Q$  to set  $P_1, \dots, P_J$  according to Eq. (2);
14  end
15  if  $k > 0$  then
16     $\varepsilon(k) = UpdateEpsilon(\tau, r_k, \alpha, \phi_{max}, T_c, k)$ 
17  end
18  for  $j \leftarrow 1$  to  $J$  do
19     $P_{j_{fea}} = \{y \in P_j | \phi(y) < \varepsilon(k)\}$ ;
20     $P_{j_{inf}} = \{y \in P_j | \phi(y) \geq \varepsilon(k)\}$ ;
21    if  $|P_j| \leq S$  then
22      randomly select  $|S - P_j|$  solutions add to  $P_j$ ;
23    end
24    if  $|P_j| > S$  then
25      if  $|P_{j_{fea}}| \geq S$  then
26        select  $S$  solutions from  $P_{j_{fea}}$  by using NSGA-II [2].
27      else if  $|P_{j_{fea}}| < S$  then
28        sort each solution in  $P_{j_{inf}}$  according to their constraint
        violations in a ascend way, and select the first  $S - |P_{j_{fea}}|$ 
        solutions and all the solutions in  $P_{j_{fea}}$  to the next generation.
29      end
30    end
31  end
32   $gen = gen + 1$ ;
33 end

```

4. Parameter setting in CM2M2: the number of infeasible weights  $N_1 = 90$ , the number of feasible weights  $N_2 = 210$ .
5. For the bi-objective CMOPs,  $J$  is set as 10 in M2M-IEpsilon, CM2M, and CM2M2. For the tri-objective CMOPs,  $J$  is set as 15 in M2M-IEpsilon, CM2M, and CM2M2. Where  $J$  is the number of sub-problems.
6. Parameter settings of IEpsilon method in M2M-IEpsilon and MOEA/D-IEpsilon:  $\theta = 0.05N$ ,  $\alpha = 0.95$ ,  $T_c = 800$ , and  $\tau = 0.1$ .
7. Parameter settings of PPS-MOEA/D could be found in Ref. [4].
8. Parameter settings of MOEA/D-CDP:  $T$  is set to 30,  $n_r$  is set to 2.

Two commonly performance metrics, including the inverted generation distance metric (IGD) [1] and the hypervolume metric (HV) [15], are employed. A set of imbalanced CMOPs with both diversity-hard and convergence-hard constraints are designed by using the DAS-CMOPs framework [5], which are named as ICD-CMOPs. The constraint functions are designed with convergence-hard and diversity-hard properties, which are generated from DAS-CMOPs [5]. The detail definition of the ICD-CMOPs can be found in the following link: <https://github.com/lwjhxx/Imbalanced-CMOPs>.

**Table 1.** The IGD results of the seven CMOEAs on ICD-CMOP1–7.

Test instance		M2M-IEpsilon	CM2M	CM2M2	NSGA-II	MOEA/D	PPS	IEpsilon
ICD-CMOP1	mean	<b>1.86E-02</b>	8.50E-02†	1.49E-01†	3.23E-01†	3.12E-01†	2.97E-01†	2.62E-01†
	std	3.43E-03	3.06E-02	3.27E-02	1.29E-02	4.87E-02	6.52E-02	7.20E-02
ICD-CMOP2	mean	<b>1.21E-01</b>	2.26E-01†	1.47E-01	3.09E-01†	2.73E-01†	2.65E-01†	2.66E-01 †
	std	8.59E-02	6.09E-02	6.21E-02	2.56E-02	7.50E-03	5.66E-03	1.60E-02
ICD-CMOP3	mean	<b>2.04E-01</b>	3.22E-01†	3.98E-01†	8.54E-01†	4.85E-01†	6.37E-01†	4.44E-01†
	std	8.40E-02	8.05E-02	6.47E-02	8.12E-02	9.17E-02	1.85E-01	9.82E-02
ICD-CMOP4	mean	<b>2.16E-02</b>	2.14E-01†	8.50E-02†	3.33E-01†	2.86E-01†	2.86E-01†	2.86E-01†
	std	5.18E-02	7.35E-02	6.05E-02	2.39E-02	1.50E-02	1.22E-02	4.31E-02
ICD-CMOP5	mean	<b>4.99E-02</b>	1.27E-01†	1.71E-01†	2.93E-01†	3.24E-01†	3.09E-01†	2.84E-01 †
	std	2.05E-02	2.97E-02	2.31E-02	4.75E-02	4.25E-03	1.44E-02	4.16E-02
ICD-CMOP6	mean	3.92E-01	4.11E-01	<b>3.15E-01</b> †	8.05E-01†	7.18E-01†	3.52E-01	7.26E-01†
	std	1.19E-01	1.18E-01	1.02E-02	9.64E-03	1.55E-02	4.74E-02	2.65E-02
ICD-CMOP7	mean	4.92E-01	5.44E-01†	5.35E-01†	7.47E-01†	7.19E-01†	<b>4.00E-01</b> †	7.24E-01 †
	std	2.78E-02	4.89E-02	2.52E-02	6.90E-03	9.53E-02	5.56E-02	6.62E-03
Wilcoxon-Test(S-D-I)	–		6-1-0	6-1-0	7-0-0	7-0-0	6-1-0	7-0-0

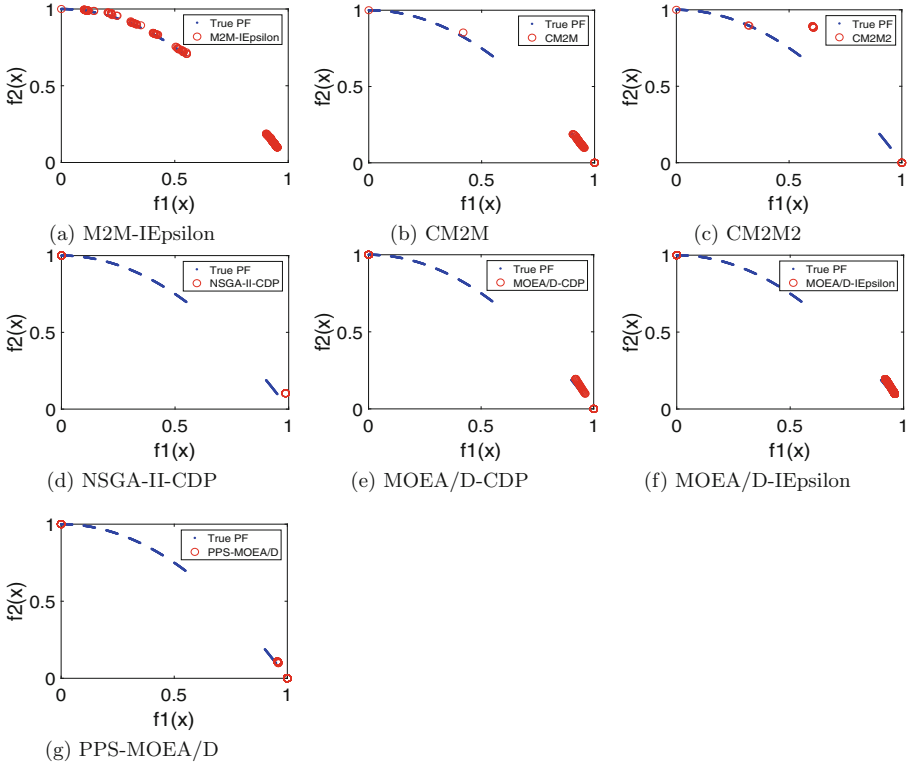
The IGD results of ICD-CMOP1-7 obtained by M2M-IEpsilon, CM2M, CM2M2, NSGA-II-CDP, MOEA/D-CDP, MOEA/D-IEpsilon and PPS-MOEA/D are presented in Table 1. The Wilcoxon-Test indicates that the proposed M2M-IEpsilon is significantly better than NSGA-II-CDP, MOEA/D-IEpsilon, and MOEA/D-CDP on ICD-CMOP1-7 test instances. For ICD-CMOP6, M2M-IEpsilon has similar performance with CM2M and PPS-MOEA/D. However, for ICD-CMOP1-5 and ICD-CMOP7, M2M-IEpsilon outperforms CM2M and PPS-MOEA/D significantly. M2M-IEpsilon is significantly better than CM2M2 on ICD-CMOPs except ICD-CMOP2.

**Table 2.** The HV results of the seven CMOEAs on ICD-CMOP1-7.

Test instance		M2M-IEpsilon	CM2M	CM2M2	NSGA-II	MOEA/D	PPS	IEpsilon
ICD-CMOP1	mean	<b>9.92E-01</b>	9.14E-01†	8.56E-01†	5.21E-01†	5.51E-01†	5.64E-01†	6.21E-01†
	std	4.34E-03	3.88E-02	1.97E-02	2.46E-02	8.77E-02	1.15E-01	1.33E-01
ICD-CMOP2	mean	<b>4.98E-01</b>	4.37E-01†	4.31E-01†	2.60E-01†	3.77E-01†	3.84E-01†	3.66E-01†
	std	5.06E-02	3.03E-02	3.35E-02	5.46E-02	1.93E-02	1.24E-02	4.24E-02
ICD-CMOP3	mean	<b>3.66E-01</b>	2.84E-01†	2.55E-01†	2.39E-01†	2.46E-01†	2.54E-01†	2.39E-01†
	std	7.72E-02	6.31E-02	2.83E-02	8.47E-17	2.59E-02	4.05E-02	8.47E-17
ICD-CMOP4	mean	<b>8.15E-01</b>	5.84E-01†	7.24E-01†	4.13E-01†	5.11E-01†	5.04E-01†	4.78E-01†
	std	7.00E-02	6.77E-02	8.05E-02	3.13E-02	1.68E-02	1.04E-02	5.83E-02
ICD-CMOP5	mean	<b>9.47E-01</b>	8.22E-01†	8.06E-01†	6.14E-01†	5.80E-01†	5.88E-01†	6.06E-01†
	std	2.35E-02	4.12E-02	1.97E-02	5.99E-02	0.00E+00	7.50E-03	5.75E-02
ICD-CMOP6	mean	1.90E-01	1.73E-01†	2.06E-01	7.09E-03†	8.99E-03†	<b>4.55E-01</b>	1.12E-02†
	std	6.17E-02	5.81E-02	9.26E-03	7.50E-04	3.96E-03	3.63E-02	1.12E-02
ICD-CMOP7	mean	3.22E-01	2.59E-01†	2.39E-01†	1.41E-01†	1.84E-01†	<b>5.61E-01</b>	1.75E-01†
	std	3.33E-02	3.57E-02	2.84E-02	7.00E-03	8.91E-03	3.24E-02	7.55E-03
Wilcoxon-Test(S-D-I)	-		7-0-0	6-1-0	7-0-0	7-0-0	5-2-0	7-0-0

Table 2 shows the HV results of ICD-CMOP1-7 obtained by the seven CMOEAs. For ICD-CMOP1-5, M2M-IEpsilon is significantly better than the rest of CMOEAs. For ICD-CMOP6, M2M-IEpsilon significantly outperforms CM2M, NSGA-II-CDP, MOEA/D-IEpsilon and MOEA/D-CDP, and has no significant difference with CM2M2 and PPS-MOEA/D. For ICD-CMOP7, M2M-IEpsilon significantly outperforms CM2M, CM2M2, NSGA-II-CDP, MOEA/D-IEpsilon and MOEA/D-CDP. The above analysis and observations reveal that the proposed algorithm M2M-IEpsilon is significantly better than the rest of CMOEAs on most of the ICD-CMOPs.

To further discuss the advantages of the proposed M2M-IEpsilon in solving ICD-CMOPs, non-dominated solutions with the median HV values achieved by seven algorithms on ICD-CMOP2 plotted in Fig. 2, the proposed M2M-IEpsilon is able to find the whole true PFs while other six CMOEAs can only converge to a part of the true PFs. There are two possible reasons for this. The first one is that the objective functions of ICD-CMOP2 are imbalanced, and only the M2M decomposition method can effectively deal with CMOPs with the imbalanced objective functions. Another reason is that ICD-CMOP2 has diversity-hard and convergence-hard constraints, which makes CM2M and CM2M2 difficult to converge to the whole true PFs, because CM2M and CM2M2 have no specific mechanisms for dealing with constraints to solve CMOPs with simultaneous diversity-hard and convergence-hard constraints. However, the proposed M2M-IEpsilon converge to disconnected PFs, which enables it to have the best performance on the ICD-CMOPs.



**Fig. 2.** The non-dominated solutions got by seven CMOEAs on ICD-CMOP2.

## 5 Conclusion

A hybrid CMOEA is proposed in the paper, namely M2M-IEpsilon, which combines the improved epsilon constraint-handling approach and M2M decomposition method to solve imbalanced CMOPs with simultaneous convergence-hard and diversity-hard constraints. A series of problems, namely ICD-CMOPs, is suggested to evaluate the performance of M2M-IEpsilon by using the DAS-CMOPs framework. The ICD-CMOPs consist of imbalanced CMOPs with diversity-hard and convergence-hard constraints. Since M2M-IEpsilon adopts the M2M decomposition method, it is able to solve CMOPs with imbalanced objective functions. The IEpsilon constraint-handling mechanism embedded in M2M-IEpsilon can help the population of M2M-IEpsilon to get across infeasible regions and to improve the diversity performance. To verify this, seven CMOEAs are tested on the ICD-CMOPs. Through the analysis of experimental results, the superiority of the proposed M2M-IEpsilon in dealing with ICD-CMOPs with simultaneous convergence-hard and diversity-hard constraints. In the future, a scheduled work is to combine machine learning techniques with M2M-IEpsilon to solve CMOPs with expensive objective and constraint functions.



## References

1. Bosman, P.A., Thierens, D.: The balance between proximity and diversity in multiobjective evolutionary algorithms. *IEEE Trans. Evol. Comput.* **7**(2), 174–188 (2003)
2. Deb, K., Pratap, A., Agarwal, S., Meyarivan, T.: A fast and elitist multiobjective genetic algorithm: NSGA-II. *IEEE Trans. Evol. Comput.* **6**(2), 182–197 (2002)
3. Fan, Z., et al.: An improved epsilon constraint-handling method in MOEA/D for CMOPs with large infeasible regions. *Soft Comput.* **23**(23), 12491–12510 (2019). <https://doi.org/10.1007/s00500-019-03794-x>
4. Fan, Z., et al.: Push and pull search for solving constrained multi-objective optimization problems. *Swarm Evol. Comput.* **44**, 665–679 (2019)
5. Fan, Z., et al.: Difficulty adjustable and scalable constrained multi-objective test problem toolkit. *Evol. Comput.* **28**(3), 339–378 (2020)
6. Fan, Z., et al.: Analysis and multi-objective optimization of a kind of teaching manipulator. *Swarm Evol. Comput.* **50**, 100554 (2019)
7. Jan, M.A., Khanum, R.A.: A study of two penalty-parameterless constraint handling techniques in the framework of MOEA/D. *Appl. Soft Comput.* **13**(1), 128–148 (2013)
8. Kalyanmoy, D., et al.: *Multi Objective Optimization Using Evolutionary Algorithms*. John Wiley and Sons, New York (2001)
9. Liu, H.L., Chen, L., Deb, K., Goodman, E.D.: Investigating the effect of imbalance between convergence and diversity in evolutionary multiobjective algorithms. *IEEE Trans. Evol. Comput.* **21**(3), 408–425 (2016)
10. Liu, H.L., Gu, F., Zhang, Q.: Decomposition of a multiobjective optimization problem into a number of simple multiobjective subproblems. *IEEE Trans. Evol. Comput.* **18**(3), 450–455 (2013)
11. Liu, H.L., Peng, C., Gu, F., Wen, J.: A constrained multi-objective evolutionary algorithm based on boundary search and archive. *Int. J. Pattern Recogn. Artif. Intell.* **30**(01), 1659002 (2016)
12. Peng, C., Liu, H.L., Gu, F.: An evolutionary algorithm with directed weights for constrained multi-objective optimization. *Appl. Soft Comput.* **60**, 613–622 (2017)
13. Wang, J., Zhou, Y., Wang, Y., Zhang, J., Chen, C.P., Zheng, Z.: Multiobjective vehicle routing problems with simultaneous delivery and pickup and time windows: formulation, instances, and algorithms. *IEEE Trans. Cybern.* **46**(3), 582–594 (2015)
14. Zhang, M., Li, H., Liu, L., Buyya, R.: An adaptive multi-objective evolutionary algorithm for constrained workflow scheduling in clouds. *Distrib. Parallel Databases* **36**(2), 339–368 (2018)
15. Zitzler, E., Thiele, L.: Multiobjective evolutionary algorithms: a comparative case study and the strength pareto approach. *IEEE Trans. Evol. Comput.* **3**(4), 257–271 (1999)