



# Distributed fixed-time attitude coordination control for multiple rigid spacecraft

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## Funding information

Shantou University (STU) Scientific Research Foundation for Talents, Grant/Award Number: NTF18015; Key Lab of Digital Signal and Image Processing of Guangdong Province; Key Laboratory of Intelligent Manufacturing Technology (Shantou University), Ministry of Education; Science and Technology Planning Project of Guangdong Province of China, Grant/Award Number: 180917144960530; Project of Educational Commission of Guangdong Province of China, Grant/Award Number: 2017KZDXM032; State Key Lab of Digital Manufacturing Equipment and Technology, Grant/Award Number: DMETKF2019020; National Defense Technology Innovation Special Zone Projects (Shantou University and National University of Defense Technology)

## Summary

In this paper, the fixed-time attitude coordination control for multiple rigid spacecraft under an undirected communication graph is investigated. By using the backstepping technique, the distributed fixed-time observer, and the method of “adding a power integrator,” a distributed fixed-time attitude coordination control law is designed for a group of spacecraft. The proposed control scheme is nonsingular and can guarantee a group of rigid spacecraft simultaneously tracking a common desired attitude within fixed time even when the time-varying reference attitude is available only to a subset of the group members. Rigorous analysis is provided to show that the attitude consensus tracking errors can converge to the origin in finite time which is bounded by a fixed constant independent of initial conditions. Numerical simulations are carried out to demonstrate the effectiveness of the proposed control law.

## KEYWORDS

adding a power integrator, attitude coordination control, fixed-time control, multiple rigid spacecraft

## 1 | INTRODUCTION

The problem of attitude coordination control for a group of rigid spacecraft has attracted considerable research attention in recent years due to its wide applications in space missions such as monitoring of the Earth and its surrounding atmosphere, geodesy, deep space imaging and exploration, and in-orbit servicing and maintenance of spacecraft. It is to be noted that most of the existing attitude coordination control laws can only achieve asymptotic stability results. In contrast to an asymptotic control law, a finite-time control law can lead to higher precision control performance, a better disturbance rejection ability, and a faster convergence rate (near the equilibrium point). Therefore, the development of a finite-time attitude coordination control scheme for multiple spacecraft is highly desirable. The design methods for the finite-time attitude coordination control problem include the terminal sliding mode method,<sup>1-3</sup> the technique of “adding a power integrator,”<sup>4,5</sup> and the homogeneity theorem.<sup>1,6-8</sup>

Terminal sliding mode control (TSMC)<sup>9,10</sup> is recognized as a finite-time control scheme. However, there exists a singularity problem in the initial TSMC. Thus, some researchers have proposed the nonsingular TSMC (NTSMC),<sup>11</sup> the modified TSMC,<sup>12</sup> and the modified fast TSMC<sup>13</sup> to eliminate the singularity problem associated with the initial TSMC. With the help of the NTSMC and the modified TSMC, the finite-time attitude coordination control problem has been studied.<sup>1-3</sup> Based on the technique of “adding a power integrator,”<sup>14</sup> a distributed finite-time attitude control law was developed in the work of Du et al<sup>4</sup> for a group of spacecraft with a leader-follower architecture, and a quaternion-based finite-time attitude synchronization control scheme was proposed in the work of Zhou et al<sup>5</sup> for a group of spacecraft. It is worth to mention that the power used in the control laws in these works<sup>4,5,14</sup> is required to be a positive odd number or a ratio of two positive odd numbers. Using the homogeneous method, Meng et al<sup>1</sup> designed a finite-time attitude containment controller for multiple rigid bodies with multiple stationary leaders. Based on the homogeneity theorem, a bounded finite-time attitude controller was proposed for multiple rigid spacecraft with a leader-follower architecture.<sup>6</sup> By using the homogeneous method, a finite-time attitude synchronization control scheme was developed for multiple spacecraft under a directed communication topology.<sup>7</sup> With the aid of the homogeneity theorem and the finite-time observer derived in Zou,<sup>15</sup> a distributed velocity-free attitude coordination control law was proposed for spacecraft formations.<sup>8</sup>

It should be emphasized that the convergent time resulted in the finite-time control schemes is heavily dependent on initial conditions. The fixed-time control can provide some required control precision in finite time whose upper bound is independent of initial conditions.<sup>16</sup> In the work of Andrieu et al,<sup>17</sup> the bilimit homogeneity is introduced and it is shown that an asymptotically stable system is fixed-time stable if it is homogeneous of negative degree in 0-limit and homogeneous of positive degree in  $\infty$ -limit. Unfortunately, it is difficult to use such an approach to estimate and adjust the settling time. Using the sliding mode control algorithm with polynomial feedbacks, Polyakov<sup>18</sup> proposed fixed-time stabilization controllers for linear plants. With the help of implicit Lyapunov function, fixed-time control laws were designed for stabilization of a chain of integrators.<sup>16</sup> Due to the coupling among agents, it is nontrivial to apply the aforementioned fixed-time control approaches to the problem of attitude coordination control of a group of spacecraft.

With the aid of sliding mode control and/or bilimit homogeneity, fixed-time consensus control for second-order multiagent systems has been studied.<sup>19-22</sup> In the work of Zuo,<sup>19</sup> by using the fixed-time sliding surface, a fixed-time consensus tracking controller was proposed for a class of multiagent systems with double integrator dynamics. Based on the fixed-time observer and the bilimit homogeneity, a velocity-free fixed-time leader-follower consensus control scheme was presented for second-order multiagent systems.<sup>20</sup> It is important to mention that the control inputs are required to transmit among agents, which leads to an algebraic loop problem in implementing the control laws presented in these works<sup>19,20</sup> when there are cycles in the communication graph. Based on the nonsingular fixed-time sliding surface, the fixed-time coordinated tracking problem for second-order integrator systems was studied by Fu and Wang.<sup>21</sup> Specifically, distributed fixed-time observers are applied to estimate both position and velocity tracking errors, and then, the fixed-time control law for a single agent can be used to design the controller.

This paper considers the problem of fixed-time attitude coordination control for a group of spacecraft under an undirected communication graph when the time-varying reference signals are available only to a subset of the team members. The modified Rodrigues parameters (MRPs) are used as the attitude representation. Based on the distributed fixed-time observer, the backstepping technique, and the idea of adding a power integrator, a novel fixed-time attitude consensus tracking control scheme is designed. It should be pointed out that the stability result reported in this paper refers to the attitude system using the MRPs-based attitude parameterizations. The main contributions of this work are stated as follows.

1. As compared with the existing results,<sup>1-8</sup> the proposed control scheme can guarantee that the attitudes of a group of spacecraft simultaneously track a time-varying reference attitude in finite time and the upper bound of this finite time is independent of initial conditions.
2. By introducing some new lemmas, the limitation on the power in the existing control schemes based on the technique of adding a power integrator<sup>4,5,14,15</sup> is relaxed in this work.
3. Different from the other works,<sup>19-23</sup> the backstepping method together with the technique of adding a power integrator are used to design the fixed-time control law, and the implementation of the proposed control scheme does not have the algebraic loop problem as existed in the work of Zuo<sup>19</sup> and Tian et al.<sup>20</sup>

The rest of this paper is organized as follows. Some useful lemmas, spacecraft attitude dynamics, and graph theory are briefly described in Section 2. In Section 3, a fixed-time attitude coordination control law is proposed for a group of rigid spacecraft. Numerical simulation results are presented in Section 4. Finally, Section 5 concludes this paper.

## 2 | BACKGROUND AND PRELIMINARIES

### 2.1 | Notations, definitions, and lemmas

The Euclidean norm of a vector or the induced norm of a matrix is denoted by  $\|\cdot\|$ . The  $n \times n$  identity matrix is defined by  $I_n$ . The maximum and minimum eigenvalues of a symmetric matrix are represented by  $\lambda_{\max}(\cdot)$  and  $\lambda_{\min}(\cdot)$ , respectively. The Kronecker product is denoted by  $\otimes$ . For  $y_i \in R^{m_i}, i = 1, \dots, n$ ,  $\text{col}(y_1, \dots, y_n) = [y_1^T, \dots, y_n^T]^T$ . For a vector  $x \in R^3$ , the skew-symmetric matrix  $x^\times \in R^{3 \times 3}$  is defined by  $x^\times = [0, -x_3, x_2; x_3, 0, -x_1; -x_2, x_1, 0]$ . Given a vector  $x \in R^n$  and  $\alpha > 0$ , denote  $\text{sig}^\alpha(x) = \text{col}(\text{sig}^\alpha(x_1), \dots, \text{sig}^\alpha(x_n))$ ,  $x^\alpha = \text{col}(x_1^\alpha, \dots, x_n^\alpha)$ , and  $\text{diag}(|x|^\alpha) = \text{diag}(|x_1|^\alpha, \dots, |x_n|^\alpha)$ , where  $\text{sig}^\alpha(x_i) = \text{sgn}(x_i)|x_i|^\alpha (i = 1, \dots, n)$ , and  $\text{sgn}(\cdot)$  denotes the signum function defined by

$$\text{sgn}(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0, \end{cases}$$

with  $x \in R$ .

**Definition 1** (See the works of Bhat and Bernstein<sup>24,25</sup>). Consider the following system:

$$\dot{x} = f(x, t), \quad f(0, t) = 0, \quad x \in \Psi \subset R^n, \quad (1)$$

where  $f : \Psi \times R^+ \mapsto R^n$  is continuous on an open neighborhood  $\Psi$  of the origin  $x = 0$ . The origin of system (1) is (locally) finite-time stable if it is Lyapunov stable and finite-time convergent in a neighborhood  $\Psi_0 \subseteq \Psi$  of the origin. The “finite-time convergence” means that, for any initial condition  $x(t_0) = x_0 \in \Psi_0$  at any given initial time  $t_0$ , there is a settling time  $T(x_0, t_0) > 0$ , such that every solution  $x(t; t_0, x_0)$  of system (1) is defined for  $t \in [t_0, t_0 + T)$ ,  $x(t; t_0, x_0) \in \Psi_0 \setminus \{0\}$ , for  $t > t_0 + T$ ,  $x(t; t_0, x_0) = 0$  and  $\lim_{t \rightarrow t_0 + T} x(t; t_0, x_0) = 0$ . When  $\Psi = \Psi_0 = R^n$ , the origin of system (1) is said to be globally finite-time stable.

**Lemma 1** (See the work of Polyakov<sup>18</sup>). Suppose that  $\Psi = R^n$  and there exists a Lyapunov function  $V(x(t))$  defined on  $R^n$  satisfying

$$\dot{V}(x(t)) \leq -(\kappa_1 V^{\rho_1}(x(t)) + \kappa_2 V^{\rho_2}(x(t)))^k, \quad (2)$$

where  $\kappa_1, \kappa_2, \rho_1, \rho_2$ , and  $k$  are some positive constants,  $\rho_1 k < 1$ , and  $\rho_2 k > 1$ , then the origin of system (1) is fixed-time stable. The settling time satisfies  $T(x(0)) \leq 1/(\kappa_1^k(1 - \rho_1 k)) + 1/(\kappa_2^k(\rho_2 k - 1))$  for any given initial condition  $x(0) \in R^n$ .

The following corollary can be easily obtained if we set  $k = 1$ .

**Corollary 1.** Suppose that there is a Lyapunov function  $V(x(t))$  defined on a neighborhood  $\Psi_1 \subseteq \Psi \subset R^n$  of the origin, and

$$\dot{V}(x(t)) \leq -\kappa_1 V^{\rho_1}(x(t)) - \kappa_2 V^{\rho_2}(x(t)), \quad (3)$$

where  $\kappa_1 > 0, \kappa_2 > 0, 0 < \rho_1 < 1$ , and  $\rho_2 > 1$  are some constants, then the origin of system (1) is locally fixed-time stable. If  $\Psi_1 \subseteq \Psi$  is any subset (arbitrarily large) of  $R^n$ , then the origin of system (1) is semiglobally fixed-time stable. The settling time satisfies  $T(x(0)) \leq 1/(\kappa_1(1 - \rho_1)) + 1/(\kappa_2(\rho_2 - 1))$  for any given initial condition  $x(0) \in \Psi_1$ .

**Lemma 2** (See the work of Zou and Kumar<sup>26</sup>). For  $x \in R, y \in R$ , and a real number  $\alpha \geq 1$ , the following inequality holds:

$$|x + y|^\alpha \leq 2^{\alpha-1} |\text{sig}^\alpha(x) + \text{sig}^\alpha(y)|. \tag{4}$$

As a consequence,  $|x - y|^\alpha \leq 2^{\alpha-1} |\text{sig}^\alpha(x) - \text{sig}^\alpha(y)|$ .

*Proof.* When  $\alpha = 1$ , it is clear that (4) is true. When  $\alpha > 1$ , consider a function

$$f(z) = 2^{\alpha-1} (\text{sig}^\alpha(z) + \text{sig}^\alpha(1 - z)) - 1, \quad z \in R. \tag{5}$$

Since  $\frac{df}{dz} = 2^{\alpha-1} \alpha (|z|^{\alpha-1} - |1 - z|^{\alpha-1})$ , we can conclude that  $df/dz = 0$  if  $z = 1/2$ ,  $df/dz < 0$  if  $z < 1/2$ , and  $df/dz > 0$  if  $z > 1/2$ . Thus, the function  $f(z)$  reaches its minimal value at  $z = 1/2$ , that is,  $f(z) \geq f(1/2) = 0$ , which implies that

$$2^{\alpha-1} (\text{sig}^\alpha(z) + \text{sig}^\alpha(1 - z)) \geq 1, \quad \forall z \in R. \tag{6}$$

When  $x + y = 0$ , it is obvious that (4) holds. When  $x + y \neq 0$ , let  $z = x/(x + y)$ . Then, it follows from (6) that

$$1 \leq 2^{\alpha-1} \left( \text{sig}^\alpha \left( \frac{x}{x + y} \right) + \text{sig}^\alpha \left( \frac{y}{x + y} \right) \right) = 2^{\alpha-1} \left| \frac{\text{sig}^\alpha(x)}{\text{sig}^\alpha(x + y)} + \frac{\text{sig}^\alpha(y)}{\text{sig}^\alpha(x + y)} \right| = 2^{\alpha-1} \frac{|\text{sig}^\alpha(x) + \text{sig}^\alpha(y)|}{|x + y|^\alpha}. \tag{7}$$

Multiplying both sides of (7) by  $|x + y|^\alpha$ , it is concluded that (4) is true. Therefore, (4) holds for  $\alpha \geq 1$  and  $\forall x \in R, y \in R$ . □

**Lemma 3** (See the work of Zou and Kumar<sup>26</sup>). For  $x \in R, y \in R$ , and a real number  $\alpha \in (0, 1]$ , the following inequality holds:

$$|\text{sig}^\alpha(x) - \text{sig}^\alpha(y)| \leq 2^{1-\alpha} |x - y|^\alpha. \tag{8}$$

*Proof.* Define  $x_1 = \text{sig}^\alpha(x)$  and  $y_1 = \text{sig}^\alpha(y)$ . By noting that  $\alpha_1 = 1/\alpha \geq 1$ , it follows from Lemma 2 that

$$|x_1 - y_1|^{\alpha_1} \leq 2^{\alpha_1-1} |\text{sig}^{\alpha_1}(x_1) - \text{sig}^{\alpha_1}(y_1)|.$$

Substituting  $x_1 = \text{sig}^\alpha(x)$  and  $y_1 = \text{sig}^\alpha(y)$  into the above equation, we obtain

$$|\text{sig}^\alpha(x) - \text{sig}^\alpha(y)|^{\alpha_1} \leq 2^{\alpha_1-1} |x - y|,$$

which implies that  $|\text{sig}^\alpha(x) - \text{sig}^\alpha(y)| \leq 2^{1-\alpha} |x - y|^\alpha$ . □

**Lemma 4** (See the works of Qian and Lin<sup>14</sup>). For any  $x \in R, y \in R, c > 0, d > 0$ , and  $\gamma > 0$ ,

$$|x|^c |y|^d \leq \frac{c\gamma |x|^{c+d}}{c + d} + \frac{d|y|^{c+d}}{\gamma^{c/d}(c + d)}.$$

**Lemma 5** (See the works of Hardy et al<sup>27</sup>). For any  $x_i \in R, i = 1, 2, \dots, n$ , and a real number  $\nu \in (0, 1]$ ,

$$\left( \sum_{i=1}^n |x_i| \right)^\nu \leq \sum_{i=1}^n |x_i|^\nu \leq n^{1-\nu} \left( \sum_{i=1}^n |x_i| \right)^\nu.$$

**Lemma 6** (See the works of Zou et al<sup>8</sup>). For any  $x_i \in \mathbb{R}, i = 1, 2, \dots, n$ , and a real number  $\nu > 1$ ,

$$\sum_{i=1}^n |x_i|^\nu \leq \left( \sum_{i=1}^n |x_i| \right)^\nu \leq n^{\nu-1} \sum_{i=1}^n |x_i|^\nu.$$

*Remark 1.* It is worth mentioning that the technique of adding a power integrator is applied to design the control laws in the other works<sup>4,5,14,15</sup> where the power used in the controllers is restricted to be a positive odd number or a ratio of two positive odd numbers. However, this restriction can be relaxed in this work by using Lemmas 2 and 3.

## 2.2 | Spacecraft attitude kinematics and dynamics

In this paper, we consider a group of  $n$  spacecraft which are modeled as rigid bodies. Using the MRPs as the attitude representation, the attitude kinematics and dynamics for the  $i$ th ( $i = 1, 2, \dots, n$ ) spacecraft are described by<sup>28</sup>

$$\dot{q}_i = T_i(q_i)\omega_i, \quad (9)$$

$$J_i \dot{\omega}_i = -\omega_i^\times J_i \omega_i + \tau_i, \quad (10)$$

where  $\omega_i \in \mathbb{R}^3$  is the angular velocity of the  $i$ th spacecraft in a body-fixed frame,  $J_i \in \mathbb{R}^{3 \times 3}$  is the inertia matrix,  $\tau_i \in \mathbb{R}^3$  is the torque control,  $q_i(t) \in \mathbb{R}^3$  denotes the MRPs<sup>29</sup> representing the spacecraft attitude with respect to an inertial frame, defined by  $q_i(t) = \rho_i(t) \tan\left(\frac{t_i(t)}{4}\right)$ ,  $t_i \in [0, 2\pi]$ rad with  $\rho_i$  and  $t_i$  being the Euler eigenaxis and eigenangle, respectively. The advantage of the MRPs-based attitude representation is that the attitude description is valid for eigenaxis rotations up to  $360^\circ$ . The Jacobian matrix  $T_i(q_i) \in \mathbb{R}^{3 \times 3}$  in (9) is given by<sup>29</sup>

$$T_i(q_i) = \frac{1}{2} \left( \frac{1 - q_i^T q_i}{2} I_3 + q_i^\times + q_i q_i^T \right).$$

By appropriate procedures and definitions, (9) and (10) can be transformed into

$$\dot{q}_i = v_i, \quad \dot{v}_i = f_i(q_i, v_i) + g_i(q_i)\tau_i, \quad (11)$$

where  $v_i = \dot{q}_i$ ,  $f_i(q_i, v_i) = -T_i \dot{P}_i \dot{q}_i - T_i J_i^{-1} (P_i \dot{q}_i)^\times J_i P_i \dot{q}_i$ ,  $P_i = T_i^{-1}(q_i)$ , and  $g_i = T_i J_i^{-1}$ .

The main objective of this work is to design a distributed control law for  $\tau_i (i = 1, 2, \dots, n)$  such that the attitude state of all spacecraft in the formation can track a time-varying reference trajectory within fixed time even when only a subset of the group members can have access to the time-varying reference signals.

It is important to mention that, although the problem of attitude coordination control for a group of rigid spacecraft is considered in this paper, the control law derived here can be directly applicable to consensus tracking control for a more general class of second-order multiagent systems, which can be written in the form of (11).

## 2.3 | Graph theory

Using the graph theory, a weighted undirected connected graph  $G = (\Psi, E, A)$  is adopted to describe the topology of the information flow among spacecraft, where  $\Psi = \{\psi_1, \psi_2, \dots, \psi_n\}$  represents the set of nodes,  $E \subseteq \Psi \times \Psi$  denotes the set of edges, and  $A = [a_{ij}] \in \mathbb{R}^{n \times n}$  is the weighted adjacency matrix of the graph  $G$  with nonnegative elements. The  $i$ th spacecraft in the group is represented by the node  $\psi_i (i = 1, 2, \dots, n)$ , and an unordered pair  $(\psi_i, \psi_j)$  denotes an edge in the graph  $G$ .  $(\psi_i, \psi_j) \in E$  if and only if there is an information exchange between the  $i$ th spacecraft and the  $j$ th spacecraft. Furthermore,  $(\psi_i, \psi_j) \in E \Leftrightarrow (\psi_j, \psi_i) \in E$  as the graph is undirected. The communication quality between the  $i$ th and the  $j$ th spacecraft is represented by the adjacency element  $a_{ij}$ , ie,  $(\psi_i, \psi_j) \in E \Leftrightarrow a_{ij} > 0$ . In this paper, it is supposed that  $a_{ij} = a_{ji}$  and  $a_{ii} = 0$ , which implies that the weighted adjacency matrix  $A$  is a symmetric matrix.

The degree matrix of the weighted graph  $G$  is defined by  $D = \text{diag}\{d_1, d_2, \dots, d_n\}$ , where  $d_i = \sum_{j=1}^n a_{ij}$  ( $i = 1, 2, \dots, n$ ). The Laplacian matrix  $L$  of the weighted graph  $G$  is  $L = D - A$ , which is a symmetric matrix. The graph  $G$  is connected if there exists a path between any two nodes  $\psi_i$  and  $\psi_j$ .

For the attitude consensus tracking control, it is assumed that there is a virtual leader, labeled as spacecraft 0 and its state is given by  $q_0 \in R^3$ , a time-varying reference trajectory for the spacecraft formations. The graph  $\bar{G}$  is applied to describe the network topology associated with the system consisting of  $n$  spacecraft and one virtual leader. Let  $B = \text{diag}\{a_{10}, a_{20}, \dots, a_{n0}\}$  denote the constant adjacency matrix associated with  $\bar{G}$ , where  $a_{i0} > 0$  ( $i = 1, 2, \dots, n$ ) if the  $i$ th spacecraft can have access to the leader, otherwise  $a_{i0} = 0$ . The graph  $\bar{G}$  is connected if there exists a path in  $\bar{G}$  from the node  $\psi_0$  (leader) to every node  $\psi_i$  ( $i = 1, 2, \dots, n$ ). The following theorem regarding the connected graph  $\bar{G}$  is stated.

**Theorem 1** (See the works of Hong et al<sup>30</sup>). *If  $\bar{G}$  is connected, then the matrix  $L + B$  associated with  $\bar{G}$  is symmetric and positive definite.*

In order to facilitate the controller design, the following assumption is given.

**Assumption 1.** The reference attitude  $q_0$  and its first two derivatives are uniformly bounded such that  $\|q_0\| \leq B_1$ ,  $\|\dot{q}_0\| \leq B_2$ , and  $\|\ddot{q}_0\| \leq B_3$ , where  $B_i > 0$  ( $i = 1, 2, 3$ ) are some known constants.

### 3 | MAIN RESULTS

In this section, we propose a distributed fixed-time attitude coordination control scheme for a group of rigid spacecraft based on the distributed fixed-time observer, the backstepping technique, and the idea of adding a power integrator.

#### 3.1 | Distributed fixed-time observer

Since the information of the virtual leader is only available to a subset of the group members, distributed observers are designed for each spacecraft in the formation to obtain an estimation of the velocity  $v_0 = \dot{q}_0$  within fixed time.

**Lemma 7.** *Suppose  $\bar{G}$  is a connected graph and consider the following distributed fixed-time observer for the  $i$ th spacecraft:*

$$\dot{p}_i = -\beta_1 \text{sig}^{\frac{1}{\alpha_1}}(\zeta_i) - \beta_2 \text{sgn}(\zeta_i) - \beta_3 \text{sig}^{\alpha_1}(\zeta_i) - \beta_4 \text{sig}^\beta(\zeta_i), \quad (12)$$

where  $i = 1, 2, \dots, n$ ,  $\zeta_i = \sum_{j=1}^n a_{ij}(p_i - p_j) + a_{i0}(p_i - v_0)$ ,  $p_i$  is an estimation of  $v_0$ ,  $\beta_1 > 0$ ,  $\beta_2 > B_3 \geq \|\dot{v}_0\|$ ,  $\beta_3 > 0$ ,  $\beta_4 > 0$ ,  $\alpha_1 = (1 + \alpha)/2$ ,  $0 < \alpha < 1$  and  $\beta > 1$  are some constants. Then, the estimation error  $\tilde{p}_i = p_i - v_0$  converges to zero within fixed time.

*Proof.* The governing dynamic equation for  $\tilde{p}_i$  is

$$\dot{\tilde{p}}_i = -\beta_1 \text{sig}^{\frac{1}{\alpha_1}}(\zeta_i) - \beta_2 \text{sgn}(\zeta_i) - \beta_3 \text{sig}^{\alpha_1}(\zeta_i) - \beta_4 \text{sig}^\beta(\zeta_i) - \dot{v}_0. \quad (13)$$

Consider the following Lyapunov function:

$$V_p(\tilde{p}) = \frac{1}{2} \tilde{p}^T M_1 \tilde{p}, \quad (14)$$

where  $\tilde{p} = \text{col}(\tilde{p}_1, \tilde{p}_2, \dots, \tilde{p}_n)$ , and  $M_1 = (L + B) \otimes I_3$ . Noting that  $\zeta_i = \sum_{j=1}^n a_{ij}(p_i - p_j) + a_{i0}(p_i - v_0) = \sum_{j=1}^n l_{ij} \tilde{p}_j + a_{i0} \tilde{p}_i$  and  $\zeta = M_1 \tilde{p}$ , where  $l_{ij}$  is the element of the graph Laplacian matrix and  $\zeta = \text{col}(\zeta_1, \zeta_2, \dots, \zeta_n)$ , the time derivative of  $V_p(\tilde{p})$  along with (13) is given by

$$\begin{aligned} \dot{V}_p &\leq -\beta_1 \zeta^T \text{sig}^{\frac{1}{\alpha_1}}(\zeta) - (\beta_2 - B_3) \sum_{i=1}^n \|\zeta_i\| - \beta_3 \zeta^T \text{sig}^{\alpha_1}(\zeta) - \beta_4 \zeta^T \text{sig}^\beta(\zeta) \\ &\leq -\beta_3 \zeta^T \text{sig}^{\alpha_1}(\zeta) - \beta_4 \zeta^T \text{sig}^\beta(\zeta) \\ &\leq -\beta_3 \|\zeta\|^{1+\alpha_1} - (3n)^{\frac{1-\beta}{2}} \beta_4 \|\zeta\|^{1+\beta}, \end{aligned} \quad (15)$$

where the following inequalities

$$-\beta_3 \zeta^T \text{sig}^{\alpha_1}(\zeta) = -\beta_3 \sum_{i=1}^n \sum_{j=1}^3 \left( \zeta_{ij}^2 \right)^{\frac{1+\alpha_1}{2}} \leq -\beta_3 \left( \sum_{i=1}^n \sum_{j=1}^3 \zeta_{ij}^2 \right)^{\frac{1+\alpha_1}{2}}$$

which is obtained by Lemma 5 and

$$-\beta_4 \zeta^T \text{sig}^{\beta}(\zeta) = -\beta_4 \sum_{i=1}^n \sum_{j=1}^3 \left( \zeta_{ij}^2 \right)^{\frac{1+\beta}{2}} \leq -(3n)^{\frac{1-\beta}{2}} \beta_4 \left( \sum_{i=1}^n \sum_{j=1}^3 \zeta_{ij}^2 \right)^{\frac{1+\beta}{2}}$$

which is derived by Lemma 6 have been used. By noting that

$$V_p = \frac{1}{2} \tilde{p}^T M_1 \tilde{p} = \frac{1}{2} \zeta^T M_1^{-1} \zeta \leq \frac{1}{2} \lambda_{\max}(M_1^{-1}) \|\zeta\|^2 = \frac{1}{2 \lambda_{\min}(M_1)} \|\zeta\|^2,$$

ie,  $-\|\zeta\| \leq -[2 \lambda_{\min}(M_1) V_p]^{1/2}$ , (15) becomes

$$\begin{aligned} \dot{V}_p &\leq -\beta_3 \|\zeta\|^{1+\alpha_1} - (3n)^{\frac{1-\beta}{2}} \beta_4 \|\zeta\|^{1+\beta} \\ &\leq -\beta_3 [2 \lambda_{\min}(M_1)]^{\frac{1+\alpha_1}{2}} V_p^{\frac{1+\alpha_1}{2}} - (3n)^{\frac{1-\beta}{2}} \beta_4 [2 \lambda_{\min}(M_1)]^{\frac{1+\beta}{2}} V_p^{\frac{1+\beta}{2}} \\ &= -b_1 V_p^{\frac{1+\alpha_1}{2}} - b_2 V_p^{\frac{1+\beta}{2}}, \end{aligned} \quad (16)$$

where  $b_1 = \beta_3 [2 \lambda_{\min}(M_1)]^{\frac{1+\alpha_1}{2}}$  and  $b_2 = (3n)^{\frac{1-\beta}{2}} \beta_4 [2 \lambda_{\min}(M_1)]^{\frac{1+\beta}{2}}$ , which implies that  $V_p$  converges to zero within fixed time. Thus, we conclude that the estimation error  $\tilde{p}_i = p_i - v_0 (i = 1, 2, \dots, n)$  converges to zero within fixed time.  $\square$

*Remark 2.* It is worth noting that distributed fixed-time observers have been proposed in the work of Fu and Wang<sup>21</sup> where distributed fixed-time observers are used to estimate both position and velocity tracking errors. Differing from the work of Fu and Wang,<sup>21</sup> distributed fixed-time observers in this work are used to estimate the reference velocity only, and an extra term  $-\beta_1 \text{sig}^{\frac{1}{\alpha_1}}(\zeta_i)$  is adopted in (12) to cancel certain nonlinear term for the stability analysis, which will be explained in the following. Moreover, the right-hand side of (13) is discontinuous, and hence, its solution is understood in a Filipov sense.<sup>31</sup>

*Remark 3.* In engineering practice, an obvious choice for initializing the distributed fixed-time observer would be to choose  $p_i(0) = 0 (i = 1, 2, \dots, n)$ . Furthermore, when  $\tilde{p} = 0$  (ie,  $\zeta = 0$ ) is achieved, we have  $\tilde{p}_i = 0$  and  $\zeta_i = 0$ . Solving  $\dot{\tilde{p}}_i = 0$  when  $\zeta_i = 0$  yields the equivalent output injection term<sup>32</sup>  $-\beta_2 \text{sgn}(\zeta_i)|_{eq} = \dot{v}_0$ , which implies that  $\dot{p}_i = \dot{v}_0$ .

### 3.2 | Design of fixed-time attitude coordination controller

The attitude consensus tracking error for the  $i$ th spacecraft is defined as  $\phi_i = \sum_{j=1}^n a_{ij}(q_i - q_j) + a_{i0}(q_i - q_0) = \sum_{j=1}^n l_{ij} e_{1j} + a_{i0} e_{1i}$ , where  $q_i - q_j$  is the formation-keeping attitude error,  $e_{1i} = q_i - q_0$  is the station-keeping attitude tracking error,  $i = 1, 2, \dots, n$ , and  $j = 1, 2, \dots, n$ . The formation-keeping error denotes the attitude state (ie, attitude and angular velocity) difference between an individual spacecraft and the other spacecraft in the formation, and the station-keeping error is the attitude state difference between an individual spacecraft and the virtual leader.

Denote  $\phi = \text{col}(\phi_1, \dots, \phi_n)$ ,  $e_1 = \text{col}(e_{11}, \dots, e_{1n})$ ,  $e_2 = \text{col}(e_{21}, \dots, e_{2n})$ ,  $\chi = \text{col}(\chi_1, \dots, \chi_n)$ ,  $p = \text{col}(p_1, \dots, p_n)$  and  $\tilde{p} = \text{col}(\tilde{p}_1, \dots, \tilde{p}_n)$ , where  $e_{2i} = v_i - v_0$  and  $\chi_i = v_i - p_i (i = 1, 2, \dots, n)$ , then we have  $\phi = M_1 e_1$  and  $\dot{\phi} = M_1 e_2 = M_1 (\chi + \tilde{p})$ . The controller design is based on the backstepping technique<sup>33</sup> and is explained as follows.

*Step 1.* Design of a virtual control law for  $\tilde{\chi}_i = \chi_i + k_1 \text{sig}^{\beta}(\phi_i) (i = 1, 2, \dots, n)$ , where  $k_1 > 0$  and  $\beta > 1$  are some constants.



Consider  $\bar{\chi}_i$  as a virtual control input for the system  $\dot{\phi} = M_1 e_2 = M_1(\chi + \bar{p})$ . A virtual control law for  $\bar{\chi}_i$  is designed as

$$\bar{\chi}_{di} = -k_2 \text{sig}^{\alpha_1}(\phi_i), \quad (17)$$

with  $k_2 > 0$  being some constant.

Consider the following Lyapunov function candidate:

$$V_1 = \frac{1}{2} \phi^T M_1^{-1} \phi, \quad (18)$$

then the time derivative of  $V_1$  is given by

$$\dot{V}_1 = \phi^T (\bar{\chi} - \bar{\chi}_d) + \phi^T \bar{p} - k_1 \phi^T \text{sig}^\beta(\phi) - k_2 \phi^T \text{sig}^{\alpha_1}(\phi), \quad (19)$$

where  $\bar{\chi}_d = \text{col}(\bar{\chi}_{d1}, \dots, \bar{\chi}_{dn})$ .

Using the following inequalities

$$\phi^T (\bar{\chi} - \bar{\chi}_d) \leq \sum_{i=1}^n \sum_{j=1}^3 2^{1-\alpha_1} |\phi_{ij}| |\xi_{ij}|^{\alpha_1} \leq \frac{2^{1-\alpha_1}}{1+\alpha_1} \phi^T \text{sig}^{\alpha_1}(\phi) + \frac{2^{1-\alpha_1} \alpha_1}{1+\alpha_1} \xi^T \text{sig}^{\alpha_1}(\xi),$$

where  $\xi_i = \text{sig}^{1/\alpha_1}(\bar{\chi}_i) - \text{sig}^{1/\alpha_1}(\bar{\chi}_{di})$ ,  $\xi = \text{col}(\xi_1, \dots, \xi_n)$ , and Lemmas 3 and 4 have been applied, and

$$\begin{aligned} \phi^T \bar{p} &\leq \sum_{i=1}^n \sum_{j=1}^3 |\phi_{ij}| |\bar{p}_{ij}| \leq \frac{1}{1+\alpha_1} \phi^T \text{sig}^{\alpha_1}(\phi) + \frac{\alpha_1}{1+\alpha_1} \sum_{i=1}^n \sum_{j=1}^3 |\bar{p}_{ij}|^{1+\frac{1}{\alpha_1}} \leq \frac{1}{1+\alpha_1} \phi^T \text{sig}^{\alpha_1}(\phi) + \frac{\alpha_1 [\lambda_{\max}(M_1^{-1})]^{1+\frac{1}{\alpha_1}}}{1+\alpha_1} (\zeta^T \zeta)^{\frac{1+\alpha_1}{2\alpha_1}} \\ &\leq \frac{1}{1+\alpha_1} \phi^T \text{sig}^{\alpha_1}(\phi) + \frac{(3n)^{\frac{1-\alpha_1}{2\alpha_1}} \alpha_1 [\lambda_{\max}(M_1^{-1})]^{1+\frac{1}{\alpha_1}}}{1+\alpha_1} \zeta^T \text{sig}^{\frac{1}{\alpha_1}}(\zeta), \end{aligned}$$

where Lemmas 4 and 6 and the fact that  $\bar{p} = M_1^{-1} \zeta$  have been applied, (19) becomes

$$\dot{V}_1 \leq - \left( k_2 - \frac{1+2^{1-\alpha_1}}{1+\alpha_1} \right) \phi^T \text{sig}^{\alpha_1}(\phi) - k_1 \phi^T \text{sig}^\beta(\phi) + \frac{2^{1-\alpha_1} \alpha_1}{1+\alpha_1} \xi^T \text{sig}^{\alpha_1}(\xi) + b_3 \zeta^T \text{sig}^{\frac{1}{\alpha_1}}(\zeta), \quad (20)$$

where  $b_3 = \frac{(3n)^{\frac{1-\alpha_1}{2\alpha_1}} \alpha_1}{(1+\alpha_1) [\lambda_{\min}(M_1)]^{\frac{1+\alpha_1}{2\alpha_1}}}$  as  $\lambda_{\max}(M_1^{-1}) = 1/\lambda_{\min}(M_1)$ . Since the term  $b_3 \zeta^T \text{sig}^{\frac{1}{\alpha_1}}(\zeta)$  in (20) is not zero before estimation errors  $\bar{p}_i (i = 1, 2, \dots, n)$  converge to zero, the term  $-\beta_1 \text{sig}^{\frac{1}{\alpha_1}}(\zeta_i)$  used in (12) is employed to cancel the term  $b_3 \zeta^T \text{sig}^{\frac{1}{\alpha_1}}(\zeta)$  for the stability analysis of the resultant closed-loop system.

**Step 2.** Design of the control law for  $\tau_i (i = 1, 2, \dots, n)$ .

The dynamic equation for  $\bar{\chi}_i$  is

$$\dot{\bar{\chi}}_i = f_i(q_i, v_i) + g_i(q_i) \tau_i - \dot{p}_i + k_1 \beta \text{diag}(|\phi_i|^{\beta-1}) \dot{\phi}_i, \quad (21)$$

then the control law for  $\tau_i$  is chosen as

$$\tau_i = g_i^{-1} \left( -f_i(q_i, v_i) - k_1 \beta \text{diag}(|\phi_i|^{\beta-1}) \dot{\phi}_i - \bar{k}_3 \text{sig}^\alpha(\xi_i) - \bar{k}_4 \text{sig}^{\beta-1+\alpha_1}(\xi_i) + \dot{p}_i \right), \quad (22)$$

where  $\bar{k}_3 = k_2^{\frac{1}{\alpha_1}} (2-\alpha_1) k_3$ ,  $k_3 > 0$ ,  $\bar{k}_4 = k_2^{\frac{1}{\alpha_1}} (2-\alpha_1) k_4$  and  $k_4 > 0$  are some constants, and  $\dot{\phi}_i = \sum_{j=1}^n a_{ij}(v_i - v_j) + a_{i0}(v_i - v_0)$ . The main result of this paper is stated in the following theorem.



**Theorem 2.** Consider a group of  $n$  spacecraft described by (9) and (10), and suppose that Assumption 1 is satisfied,  $\bar{G}$  is a connected graph, and there is no singularity for all initial attitudes. If the control law is given by (22), then there exist the design parameters  $k_i$  and  $\beta_i$  ( $i = 1, 2, 3, 4$ ) such that the station-keeping attitude state tracking errors  $e_1$  and  $e_2$  converge to the origin within fixed time.

*Proof.* Consider the following Lyapunov function candidate:

$$V_2 = \frac{1}{(2 - \alpha_1)k_2^{\frac{1}{\alpha_1}}} \sum_{i=1}^n V_{2i}, \quad (23)$$

where  $V_{2i} = \sum_{j=1}^3 V_{2i,j}$ , and  $V_{2i,j}$  is defined by

$$V_{2i,j} = \int_{\bar{\chi}_{dij}}^{\bar{\chi}_{ij}} \text{sig}^{2-\alpha_1} \left( \text{sig}^{\frac{1}{\alpha_1}}(s) - \text{sig}^{\frac{1}{\alpha_1}}(\bar{\chi}_{dij}) \right) ds. \quad (24)$$

Similar to the proof of Propositions B.1 and B.2 in the work of Qian and Lin,<sup>14</sup> we can obtain that  $V_{2i,j}$  ( $i = 1, 2, \dots, n, j = 1, 2, 3$ ) is differentiable, positive definite, and proper.

The time derivative of  $V_{2i,j}$  ( $i = 1, 2, \dots, n, j = 1, 2, 3$ ) can be derived as

$$\dot{V}_{2i,j} = \frac{1}{(2 - \alpha_1)k_2^{\frac{1}{\alpha_1}}} \text{sig}^{2-\alpha_1}(\xi_{ij}) \dot{\bar{\chi}}_{ij} + \dot{\phi}_{ij} \int_{\bar{\chi}_{dij}}^{\bar{\chi}_{ij}} \text{sig}^{1-\alpha_1} \left( \text{sig}^{\frac{1}{\alpha_1}}(s) - \text{sig}^{\frac{1}{\alpha_1}}(\bar{\chi}_{dij}) \right) ds. \quad (25)$$

Substituting the control law (22) into (21) and using (25), the time derivative of  $V_2$  is

$$\begin{aligned} \dot{V}_2 &= -k_3 \xi^T \text{sig}^{\alpha_1}(\xi) - k_4 \xi^T \text{sig}^\beta(\xi) + \sum_{i=1}^n \sum_{j=1}^3 \dot{\phi}_{ij} \Pi_{ij} \\ &= -k_3 \xi^T \text{sig}^{\alpha_1}(\xi) - k_4 \xi^T \text{sig}^\beta(\xi) + (M_1 e_2)^T \Pi \\ &\leq \lambda_{\max}(M_1) \|\Pi\| \|\bar{\chi} - \bar{\chi}_d - k_1 \text{sig}^\beta(\phi) - k_2 \text{sig}^{\alpha_1}(\phi) + \tilde{p}\| - k_3 \xi^T \text{sig}^{\alpha_1}(\xi) - k_4 \xi^T \text{sig}^\beta(\xi), \end{aligned} \quad (26)$$

where  $\Pi_{ij} = \int_{\bar{\chi}_{dij}}^{\bar{\chi}_{ij}} \text{sig}^{1-\alpha_1} \left( \text{sig}^{\frac{1}{\alpha_1}}(s) - \text{sig}^{\frac{1}{\alpha_1}}(\bar{\chi}_{dij}) \right) ds$ ,  $i = 1, \dots, n, j = 1, 2, 3$ ,  $\Pi \in \mathbb{R}^{3n}$ , and the fact that  $e_2 = \chi + \tilde{p} = \bar{\chi} - \bar{\chi}_d - k_1 \text{sig}^\beta(\phi) - k_2 \text{sig}^{\alpha_1}(\phi) + \tilde{p}$  has been applied. Noting that

$$\begin{aligned} \|\text{sig}^{\alpha_1}(\phi)\| &\leq (3n)^{\frac{1-\alpha_1}{2}} \|\phi\|^{\alpha_1} \\ \|\text{sig}^\beta(\phi)\| &\leq \|\phi\|^\beta \\ \|\bar{\chi} - \bar{\chi}_d\| &\leq \sum_{i=1}^n \sum_{j=1}^3 |\bar{\chi}_{ij} - \bar{\chi}_{dij}| \leq 2^{1-\alpha_1} \sum_{i=1}^n \sum_{j=1}^3 |\xi_{ij}|^{\alpha_1} \leq 2^{1-\alpha_1} (3n)^{\frac{1-\alpha_1}{2}} \|\xi\|^{\alpha_1} \\ \|\Pi\| &\leq \sqrt{\sum_{i=1}^n \sum_{j=1}^3 |\bar{\chi}_{ij} - \bar{\chi}_{dij}|^2 |\xi_{ij}|^{2(1-\alpha_1)}} \leq 2^{1-\alpha_1} \|\xi\|, \end{aligned}$$

we have

$$\begin{aligned} (M_1 e_2)^T \Pi &\leq 2^{2-2\alpha_1} (3n)^{\frac{1-\alpha_1}{2}} \lambda_{\max}(M_1) \|\xi\|^{1+\alpha_1} + 2^{1-\alpha_1} k_1 \lambda_{\max}(M_1) \|\xi\| \|\phi\|^\beta + 2^{1-\alpha_1} (3n)^{\frac{1-\alpha_1}{2}} k_2 \lambda_{\max}(M_1) \|\xi\| \|\phi\|^{\alpha_1} \\ &\quad + 2^{1-\alpha_1} \lambda_{\max}(M_1) \|\xi\| \|\tilde{p}\| \\ &\leq c_2 \|\xi\|^{1+\alpha_1} + \frac{\|\xi\|^{1+\beta}}{1+\beta} + c_3 \|\phi\|^{1+\beta} + \frac{2\|\xi\|^{1+\alpha_1}}{1+\alpha_1} + c_4 \|\phi\|^{1+\alpha_1} + \frac{\alpha_1 c_1^{\frac{1}{\alpha_1}}}{1+\alpha_1} \|\tilde{p}\|^{1+\frac{1}{\alpha_1}} \\ &\leq \left( c_2 + \frac{2}{1+\alpha_1} \right) \xi^T \text{sig}^{\alpha_1}(\xi) + \frac{(3n)^{\frac{\beta-1}{2}}}{1+\beta} \xi^T \text{sig}^\beta(\xi) + c_3 (3n)^{\frac{\beta-1}{2}} \phi^T \text{sig}^\beta(\phi) + c_4 \phi^T \text{sig}^{\alpha_1}(\phi) + c_5 \zeta^T \text{sig}^{\frac{1}{\alpha_1}}(\zeta), \end{aligned}$$

where  $c_1 = 2^{1-\alpha_1} \lambda_{\max}(M_1)$ ,  $c_2 = 2^{1-\alpha_1} (3n)^{\frac{1-\alpha_1}{2}}$ ,  $c_3 = \frac{\beta(2^{1-\alpha_1} k_1 c_1)^{\frac{1}{\beta}}}{1+\beta}$ ,  $c_4 = \frac{\alpha_1 \left[ k_2 c_1 (3n)^{\frac{1-\alpha_1}{2}} \right]^{\frac{1}{\alpha_1}}}{1+\alpha_1}$ , and  $c_5 = \frac{\alpha_1 c_1^{\frac{1}{\alpha_1}} (3n)^{\frac{1-\alpha_1}{2\alpha_1}}}{(1+\alpha_1)(\lambda_{\min}(M_1))^{1+\frac{1}{\alpha_1}}}$ .

Then, (26) becomes

$$\begin{aligned} \dot{V}_2 \leq & - \left( k_3 - c_2 - \frac{2}{1+\alpha_1} \right) \xi^T \text{sig}^{\alpha_1}(\xi) - \left( k_4 - \frac{(3n)^{\frac{\beta-1}{2}}}{1+\beta} \right) \xi^T \text{sig}^{\beta}(\xi) + c_3 (3n)^{\frac{\beta-1}{2}} \phi^T \text{sig}^{\beta}(\phi) \\ & + c_4 \phi^T \text{sig}^{\alpha_1}(\phi) + c_5 \zeta^T \text{sig}^{\frac{1}{\alpha_1}}(\zeta). \end{aligned} \quad (27)$$

Consider the following Lyapunov function:

$$V = K_1 V_1 + V_2 + K_2 V_p, \quad (28)$$

where  $K_1 = \max\{(3n)^{\frac{\beta-1}{2}} c_3, c_4\}$ ,  $K_2 = K_1 b_3 + c_5$ ,  $V_1$ ,  $V_2$ , and  $V_p$  are defined in (18), (23), and (14), respectively. Using (15), (20), and (27), the time derivative of  $V$  is given by

$$\begin{aligned} \dot{V} \leq & -K_1 \left( k_2 - 1 - \frac{1+2^{1-\alpha_1}}{1+\alpha_1} \right) \phi^T \text{sig}^{\alpha_1}(\phi) - K_1 (k_1 - 1) \phi^T \text{sig}^{\beta}(\phi) - \left( k_4 - \frac{(3n)^{\frac{\beta-1}{2}}}{1+\beta} \right) \xi^T \text{sig}^{\beta}(\xi) \\ & - \left( k_3 - c_2 - \frac{2+K_1 \alpha_1 2^{1-\alpha_1}}{1+\alpha_1} \right) \xi^T \text{sig}^{\alpha_1}(\xi) - K_2 (\beta_1 - 1) \zeta^T \text{sig}^{\frac{1}{\alpha_1}}(\zeta) - K_2 \beta_3 \zeta^T \text{sig}^{\alpha_1}(\zeta) - K_2 \beta_4 \zeta^T \text{sig}^{\beta}(\zeta). \end{aligned} \quad (29)$$

If we choose the control parameters such that  $k_2 > 1 + \frac{1+2^{1-\alpha_1}}{1+\alpha_1} + k$ ,  $k_1 > 1 + k$ ,  $k_4 > \frac{(3n)^{\frac{\beta-1}{2}}}{1+\beta} + k$ ,  $k_3 > c_2 + \frac{2+K_1 \alpha_1 2^{1-\alpha_1}}{1+\alpha_1} + k$  and  $\beta_1 > 1$ , where  $k > 0$  is some constant, then we have

$$\begin{aligned} \dot{V} \leq & -K_1 k \phi^T \text{sig}^{\alpha_1}(\phi) - K_1 k \phi^T \text{sig}^{\beta}(\phi) - k \xi^T \text{sig}^{\alpha_1}(\xi) - k \xi^T \text{sig}^{\beta}(\xi) - K_2 \beta_3 \zeta^T \text{sig}^{\alpha_1}(\zeta) - K_2 \beta_4 \zeta^T \text{sig}^{\beta}(\zeta) \\ \leq & -\kappa \|e\|^{1+\alpha_1} - (3n)^{\frac{1-\beta}{2}} \kappa \|e\|^{1+\beta}, \end{aligned} \quad (30)$$

where  $\kappa = \min\{K_1 k, k, K_2 \beta_3, K_2 \beta_4\}$ ,  $e = \text{col}(\phi, \xi, \zeta)$ , and Lemmas 5 and 6 have been applied. Noting that

$$\begin{aligned} V & \leq \frac{K_1 \lambda_{\max}(M_1^{-1})}{2} \|\phi\|^2 + \frac{2^{1-\alpha_1}}{(2-\alpha_1)k_2^{\frac{1}{\alpha_1}}} \|\xi\|^2 + \frac{K_2}{\lambda_{\min}(M_1)} \|\zeta\|^2 \\ & \leq K \|e\|^2, \end{aligned}$$

where

$$K = \max \left\{ \frac{K_1 \lambda_{\max}(M_1^{-1})}{2}, \frac{2^{1-\alpha_1}}{(2-\alpha_1)k_2^{\frac{1}{\alpha_1}}}, \frac{K_2}{\lambda_{\min}(M_1)} \right\},$$

the time derivative of  $V$  in (30) becomes

$$\dot{V} \leq -\frac{\kappa}{K^{\frac{1+\alpha_1}{2}}} V^{\frac{1+\alpha_1}{2}} - \frac{(3n)^{\frac{1-\beta}{2}} \kappa}{K^{\frac{1+\beta}{2}}} V^{\frac{1+\beta}{2}}. \quad (31)$$

By Corollary 1, we can conclude that  $\phi$ ,  $\xi$ , and  $\zeta$  converge to zero within fixed time, which in turn implies that  $\bar{\chi}$  and  $\bar{\chi}_d$  converge to zero within fixed time as well. Since  $e_1 = M_1 \phi$  and  $e_2 = \bar{\chi} - \bar{\chi}_d - k_1 \text{sig}^{\beta}(\phi) - k_2 \text{sig}^{\alpha_1}(\phi) + M_1^{-1} \zeta$ , we can obtain that the station-keeping attitude state tracking errors  $e_1$  and  $e_2$  converge to the origin within fixed time.  $\square$

*Remark 4.* Note that the proposed control scheme (22) depends only on the information of its own and its neighbors, and thus, it is a distributed control law. Furthermore, the control law (22) is discontinuous since a discontinuous term  $\dot{p}_i (i = 1, 2, \dots, n)$  is included. In practical applications, the discontinuous function  $\text{sgn}(x)$  is replaced by a smooth function  $\tanh(x/\epsilon)$ , where  $x \in R^n$ , and  $\epsilon$  is a small positive constant. In this case, the station-keeping tracking errors  $e_1$  and  $e_2$  will converge to a small bounded region within fixed-time rather than approach to the origin within fixed time.

*Remark 5.* Using the fixed-time sliding surface and the bilimit homogeneity, fixed-time consensus controllers were proposed for second-order multiagent systems in the works of Zuo<sup>19</sup> and Tian et al.<sup>20</sup> However, there exists an algebraic loop problem in implementing the controllers developed in the works of Zuo<sup>19</sup> and Tian et al.<sup>20</sup> because the control inputs are required to transmit among agents. Different from the works of Zuo<sup>19</sup> and Tian et al.,<sup>20</sup> the backstepping method and the technique of adding a power integrator are applied to design the fixed-time control law in this paper, and the algebraic loop problem existing in the works of Zuo<sup>19</sup> and Tian et al.<sup>20</sup> is not necessary in the proposed scheme.

*Remark 6.* It is to be noted that the symmetric property of the matrix  $L + B$  is employed to derive the control law presented in this paper. Since the matrix  $L + B$  may not have the symmetric property if the communication graph is directed, the proposed control law cannot be applied to the case when the communication graph is directed. The fixed-time attitude coordination controller design for the directed graph case will be one of our future works.

*Remark 7.* In this paper, adding a power integrator method is applied to design the fixed-time control law. In the literature, the sliding mode control and the bihomogeneity property can also be used to design a fixed-time control scheme. In contrast to the sliding mode control and the bihomogeneity property, the main advantage of the idea of adding a power integrator lies in the fact that it may be extendable to the case in which the system does not have the properties of feedback linearizability and linearity of the control input. Furthermore, the technique of adding a power integrator is a generalization of the integrator backstepping, and it focuses on ways to exploit the dominant nonlinearities of the dynamic system in the feedback design.<sup>34</sup> By using this design technique and Lemma 4, certain nonlinear terms in (19) and (26) can be dominated in (30) rather than be canceled.

*Remark 8.* Note that if we choose the design parameters  $\alpha = \beta = 1$ , then the controller (22) reduces to an asymptotic control law.

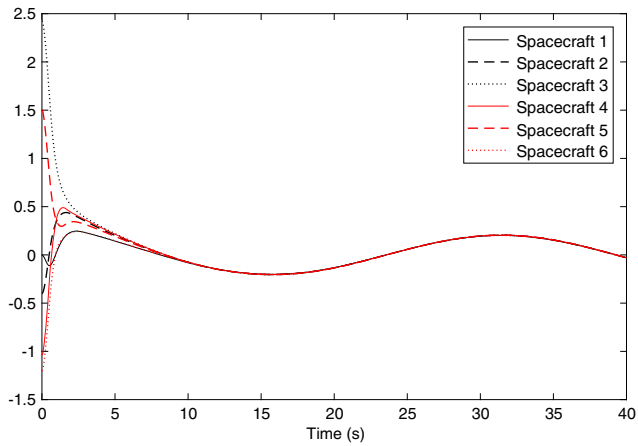
## 4 | NUMERICAL SIMULATIONS

In this section, numerical simulations of the governing nonlinear system equations of motion (ie, the spacecraft attitude system (9) and (10), the fixed-time observer (12), and the control law (22)) are conducted to illustrate the effectiveness of the proposed control scheme.

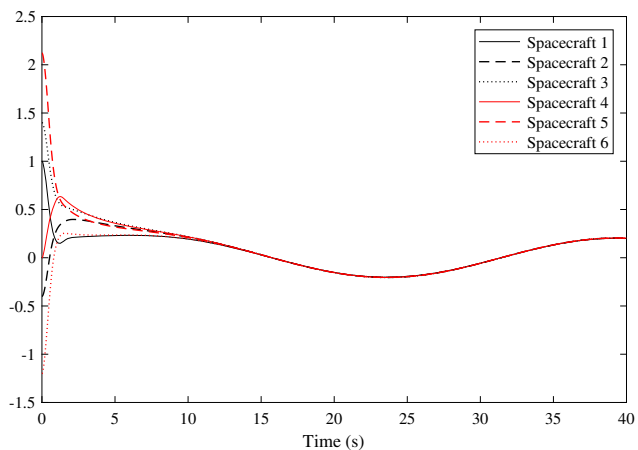
In the simulation, a group of six spacecrafts together with one virtual leader are considered. The inertia matrices are chosen the same as those in Table 1. Matrix  $A = (a_{ij})_{6 \times 6}$  is selected as  $a_{12} = a_{21} = 0.2$ ,  $a_{16} = a_{61} = 0.3$ ,  $a_{23} = a_{32} = 0.3$ ,  $a_{34} = a_{43} = 0.3$ ,  $a_{45} = a_{54} = 0.4$ ,  $a_{56} = a_{65} = 0.6$ , and  $a_{ij} = 0$  for other elements. Matrix  $B$  is taken as  $B = \text{diag}\{0.4, 0, 0, 0, 0, 0.4\}$ . The initial attitudes are  $q_1(0) = \text{col}(0, 1, \sqrt{3})$ ,  $q_2(0) = -0.4\text{col}(1, 1, \sqrt{2})$ ,  $q_3(0) = 1.4\text{col}(\sqrt{3}, 1, 0)$ ,  $q_4(0) = -0.6\text{col}(\sqrt{3}, 0, 1)$ ,  $q_5(0) = 1.5\text{col}(1, \sqrt{2}, 1)$ , and  $q_6(0) = -1.2\text{col}(1, 1, \sqrt{2})$ . The initial angular velocities are  $\omega_i(0) = 0 (i = 1, 2, \dots, 6)$ . The initial values of  $p_i (i = 1, 2, \dots, 6)$  were randomly chosen in the interval  $[-1, 1]$ . The time-varying reference attitude is assumed to be  $q_0 = 0.2\text{col}(\cos(0.2t), \sin(0.2t), \sqrt{3})$ . The controller and observer parameters are chosen as  $\alpha = 0.4$ ,  $\beta = 1.1$ ,  $k_1 = 1.1$ ,  $k_2 = 1.1$ ,  $k_3 = 2$ ,  $\epsilon = 0.01$ ,  $\beta_1 = 1.5$ ,  $\beta_2 = 0.2$ , and  $\beta_3 = \beta_4 = 1$ .

**TABLE 1** Mass moment of inertia tensor<sup>35</sup>

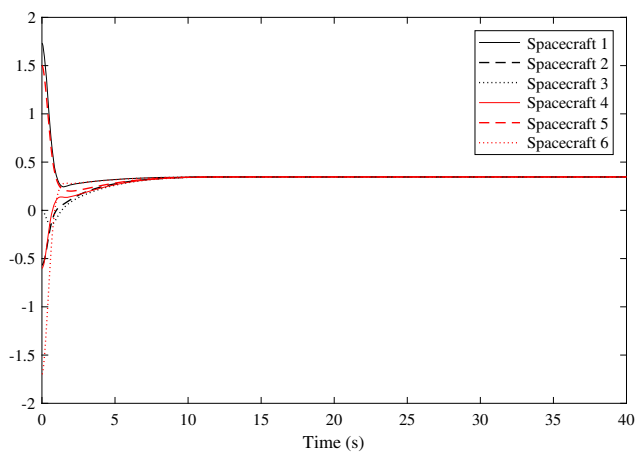
$J_1$	[10.10.1; 0.10.10.1; 0.10.10.9] kg·m <sup>2</sup>
$J_2$	[1.50.20.3; 0.20.90.4; 0.30.42.0] kg·m <sup>2</sup>
$J_3$	[0.80.10.2; 0.10.70.3; 0.20.31.1] kg·m <sup>2</sup>
$J_4$	[1.20.30.7; 0.30.90.2; 0.70.21.4] kg·m <sup>2</sup>
$J_5$	[0.90.150.3; 0.151.20.4; 0.30.41.2] kg·m <sup>2</sup>
$J_6$	[1.10.350.45; 0.351.00.5; 0.450.51.3] kg·m <sup>2</sup>



(A)



(B)

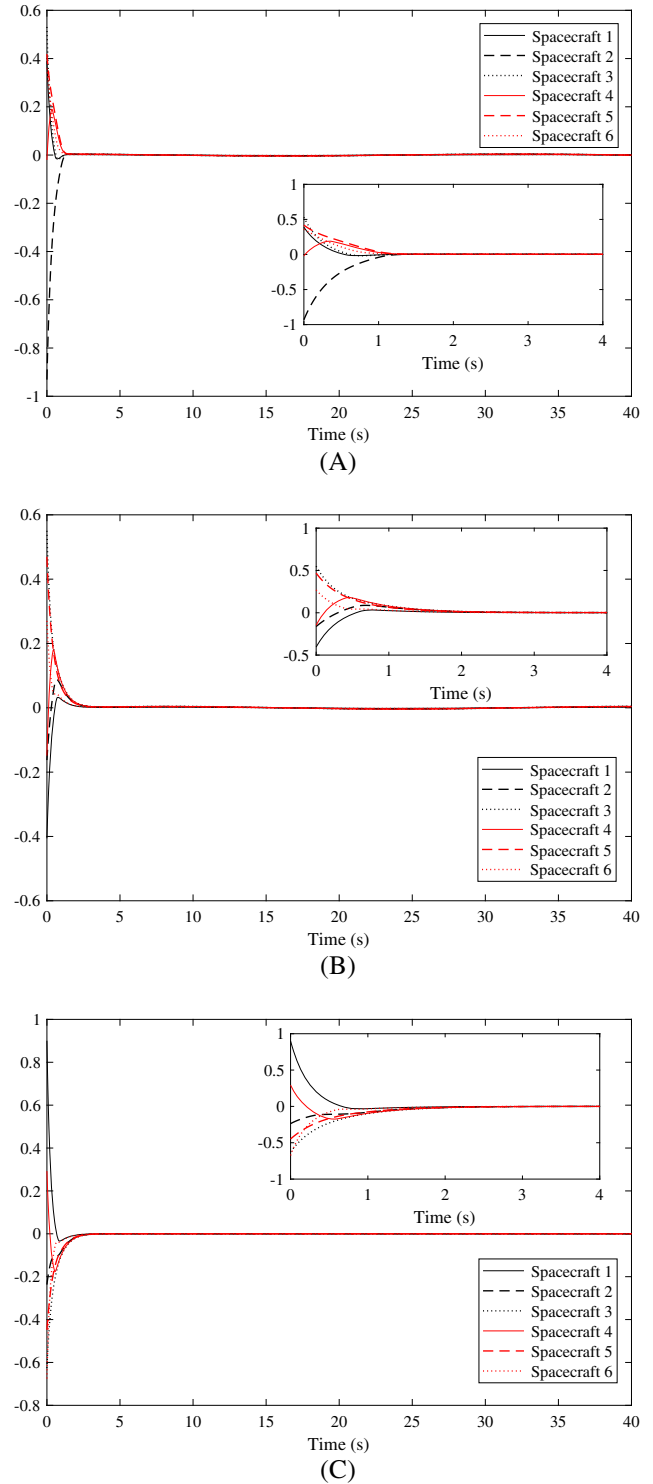


(C)

**FIGURE 1** Spacecraft attitudes. A, Response of  $q_{i1}$  ( $i = 1, 2, \dots, 6$ ); B, Response of  $q_{i2}$  ( $i = 1, 2, \dots, 6$ ); C, Response of  $q_{i3}$  ( $i = 1, 2, \dots, 6$ ) [Colour figure can be viewed at [wileyonlinelibrary.com](http://wileyonlinelibrary.com)]

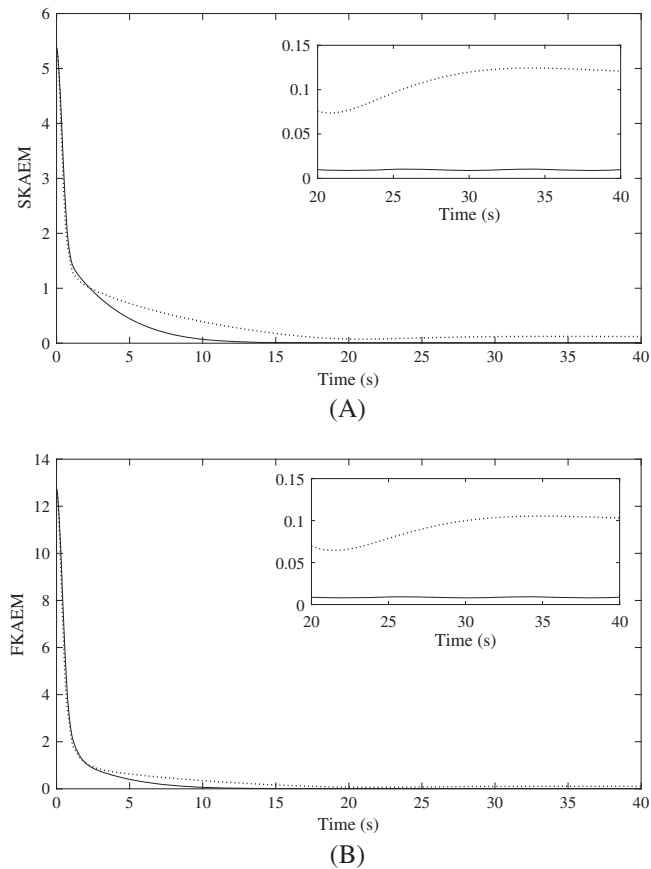
With the control law (22), the spacecraft attitudes are given in Figure 1. It can be observed that the attitudes of all spacecraft reach on a common attitude even when only a subset of the group members can have access to the reference attitude. The convergence of the distributed fixed-time observer errors  $\tilde{p}_i$  ( $i = 1, 2, \dots, n$ ) is shown in Figure 2.

Next, the performance of the fixed-time control law (22) is compared with an asymptotic control law (ie, the control law (22) with  $\alpha = 1$  and  $\beta = 1$ ). In order to investigate the performance of the control scheme, two metrics, ie,



**FIGURE 2** Distributed observer errors. A,  $\tilde{p}_{11}(i = 1, 2, \dots, 6)$ ; B,  $\tilde{p}_{12}(i = 1, 2, \dots, 6)$ ; C,  $\tilde{p}_{13}(i = 1, 2, \dots, 6)$  [Colour figure can be viewed at [wileyonlinelibrary.com](http://wileyonlinelibrary.com)]

the station-keeping attitude error metric (SKAEM) and the formation-keeping attitude error metric (FKAEM), are used, where  $SKAEM = \sqrt{\sum_{i=1}^n \|e_{1i}\|^2}$  and  $FKAEM = \sqrt{\sum_{i=1}^{n-1} \sum_{j=i+1}^n \|q_i - q_j\|^2}$ . The SKAEM and FKAEM are depicted in Figure 3. It is observed that the proposed fixed-time attitude control scheme can produce a better control performance than the asymptotic controller.



**FIGURE 3** Performance comparison between the fixed-time control law (22) and an asymptotic control law. A, Response of station-keeping attitude error metric (SKAEM) by the fixed-time controller (solid line) and the asymptotic controller (dotted line); B, Response of formation-keeping attitude error metric (FKAEM) by the fixed-time controller (solid line) and the asymptotic controller (dotted line)

## 5 | CONCLUSIONS

With the use of the distributed fixed-time observer, the backstepping technique, and the method of “adding a power integrator,” a distributed attitude coordination control scheme was designed for spacecraft formations under an undirected communication graph. The control law derived here can guarantee that the station-keeping and formation-keeping attitude state errors converge to the origin within fixed time even when the time-varying reference attitude is accessible to only a part of the team members. Numerical comparison between the fixed-time controller and an asymptotic controller was carried out to demonstrate the performance of the proposed control scheme. It was shown that the proposed controller can achieve a good attitude coordination performance even when there is only a part of the spacecraft in the formation accessible to the leader and the fixed-time control scheme can provide faster convergence rates and higher accuracies than an asymptotic control law.

## FINANCIAL DISCLOSURE

This work was supported by Shantou University (STU) Scientific Research Foundation for Talents (NTF18015), by the Key Lab of Digital Signal and Image Processing of Guangdong Province, by the Key Laboratory of Intelligent Manufacturing Technology (Shantou University), Ministry of Education, by the Science and Technology Planning Project of Guangdong Province of China under grant 180917144960530, by the Project of Educational Commission of Guangdong Province of China under grant 2017KZDXM032, by the State Key Lab of Digital Manufacturing Equipment and Technology under grant DMETKF2019020, and by the National Defense Technology Innovation Special Zone Projects (Shantou University and National University of Defense Technology).

## CONFLICT OF INTEREST

The authors declare no potential conflict of interests.

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**How to cite this article:** Zou A-M, Fan Z. Distributed fixed-time attitude coordination control for multiple rigid spacecraft. *Int J Robust Nonlinear Control*. 2020;30:266–281. <https://doi.org/10.1002/rnc.4763>