Evolution of Controllers under a Generalized Structure Encoding/Decoding Scheme with Application to Magnetic Levitation System

Bin Xin, Member, IEEE, Yipeng Wang, Wenchao Xue, Member, IEEE, Tao Cai, Zhun Fan, Senior Member, IEEE, Jiaoyang Zhan, and Jie Chen, Fellow, IEEE

Abstract—Evolutionary search has been widely implemented for the adjustment of controllers’ parameters. Nevertheless, the structure of controllers, which has a more important role in control systems, has been seldom studied. To this end, an evolutionary design method of controllers is proposed to optimize both structures and parameters simultaneously in this paper. A controller is made up of a combination of some basic controller components and relevant parameters. The design of controllers can be transformed into an optimization problem involving the structure (represented by discrete vectors) and parameters (represented by real numbers). A generalized structure encoding/decoding scheme is developed. Guided by the performance indicators, intelligent algorithms for both combinatorial and numerical optimization are employed to iteratively and cooperatively evolve the controller structure and parameters, respectively. In order to effectively reduce some redundant or infeasible solutions, a set of generation rules for the controller structure are put forward, which also ensures the feasibility of the structure. Furthermore, this method is applied to a magnetic levitation ball system with nonlinear dynamics and external disturbance. Both simulation and experiment results demonstrate the effectiveness and practicability of the proposed method.

Index Terms—Automatic design of controllers, mixed-variable optimization, intelligent optimization algorithms, disturbance-rejection, magnetic levitation system.

I. INTRODUCTION

This work was supported in part by the National Outstanding Youth Talents Support Program under Grant 61822304; in part by Beijing Advanced Innovation Center for Intelligent Robots and Systems under Grant 2019RJS09; in part by the Basic Science Center Programs of NSFC under Grant 62088101; and in part by the National Nature Science Foundation of China under Grant 61973299. (Corresponding author: Tao Cai and Bin Xin.)

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THE work environment, control objects, and tasks of production fields have become more and more complex, which makes control requirements of the industrial production increasingly demanding. As a consequence, the controller design has long been one of the hot topics in the field of control engineering and a flood of control methods of different characteristics have arisen. Admittedly, the Proportion-Integration-Differentiation (PID) control is widely used in many automatic control processes of machinery manufacturing, metallurgy, petroleum chemical engineering, and other industries, owing to its easy-to-understand principle, simple structure, and mature parameter tuning methods [1]–[3].

For better control performance, parameter tuning is inevitable before a PID controller is applied and substantial research efforts have been devoted to this matter. Existing parameter tuning methods can be classified into two categories: conventional methods and optimization-based methods. The Ziegler-Nichols method [4], the Cohen-Coon technique [5], the relay auto-tuning method [6], and the internal model control based method [7] are among the most representative conventional tuning methods, which are highly dependent on human beings and time-consuming. Thus, many intelligent methods such as genetic algorithm [8], differential evolution (DE) algorithm [9], ant colony optimization algorithm [10], and particle swarm optimization algorithm [11] have been adopted to tune PID parameters.

Parameter tuning is an arduous process and the control performance may deteriorate, especially when applied to complex, non-linear objects whose properties are difficult to describe accurately. Therefore, a series of advanced control methods for complex plants have been proposed, including adaptive control [12], robust control [13], sliding mode control (SMC) [14], active disturbance rejection control (ADRC) [15]–[18], optimal control [19], predictive control [20], fuzzy control [21], neural network control [22], and so on.

However, designers have to analyze the characteristics of objects deeply to design specific controllers using the above methods for satisfactory expected performance indicators. In ADRC, designers need to design an appropriate extended state observer to transform the nonlinear uncertain object into a integral-chain system [16]. Besides, it is a tedious task to tune numerous parameters of ADRC. As for SMC, the design of appropriate sliding surface and control law specific to the object is crucial to the performance of SMC controllers.
A number of independent simulations and experiments are to a magnetic levitation system with external disturbance. To validate the effectiveness of this method, it is applied time-consuming. In this sense, we can only rely on very system model, which makes it become a general method but evaluation in the proposed method is independent of the chance of finding feasible solutions. The simulation-based validates the difficulty of the optimization of controller, reducing operators/algorithms to search for controllers with satisfactory parameters. On the other hand, it is evident that the optimized structure can greatly improve the performance of a controller. Thus, how to achieve the optimization of both the structure and parameters during the design process becomes an important and open problem.

In the previous research, Koza et al. [24] put forward an automatic design method of control laws by genetic programming. From the view of signal processing, the controller is decomposed into several signal processing modules. The type of signal processing modules, the topology and control parameters of the modules are determined by genetic programming. Taking a 3-Lag plant with 5s delay as an example, the topology and parameters of the controller were designed by genetic programming on a cluster of 1000 computers. The simulation results showed that the obtained controller has good control effect. However, the implementation of this method depends on a large cluster of computers, the entire process will consume a lot of resources and time, which cannot meet the design requirements.

With the rapid improvement of computer performance, Xin et al. realized the simultaneous design of the structure and parameters by evolutionary algorithms [25]. The obtained controllers were denoted as the structural optimization controller (SOC). Series-parallel connection relationships between components were considered and a binary encoding method was proposed to denote the structure. A bi-level optimization strategy was proposed to optimize the structure and parameters of controllers using two variants of DE algorithms respectively. However, many structure forms are not considered and the bi-level optimization strategy is time-consuming.

In this paper, the automatic design and optimization of controllers is achieved by evolution under a generalized structure encoding/decoding scheme. The structure and parameters of controllers are optimized simultaneously using competent operators/algorithms to search for controllers with satisfactory indicators. The property of the coupled mixed-variable aggravates the difficulty of the optimization of controller, reducing the chance of finding feasible solutions. The simulation-based evaluation in the proposed method is independent of the system model, which makes it become a general method but time-consuming. In this sense, we can only rely on very limited evaluations to find satisfactory desirable controllers. To validate the effectiveness of this method, it is applied to a magnetic levitation system with external disturbance. A number of independent simulations and experiments are carried out and the results have demonstrated the effectiveness and practicability of the proposed method. For simplicity, the controller determined by the method with optimized structure and parameters is named COSP. The main contributions of this paper can be summarized as follows:

- A general automatic controller design method is proposed to achieve the optimization of expected performance indicators via flexibly combining basic controller components and tuning their parameters. The design of controllers is transformed into a mixed-variable optimization problem involving continuous and discrete decision variables. Driven by performance indicators, the structure and parameters are evolved iteratively and cooperatively using competent evolutionary algorithms respectively. Compared with traditional controller design methods, this method can not only realize the automatic tuning of parameters, but also search for a wealth of controller structures.

- Compared to the previous work [25], a generalized encoding/decoding scheme, which considers more abundant controller structures, is proposed to represent controller solutions incorporating integer and real-valued vectors. Integer vectors are employed to represent the connection relationship of the given components and real-valued vectors are adopted to represent the corresponding parameters of the given components. The proposed encoding/decoding scheme facilitates the application of intelligent optimizers in searching for high-quality solutions.

- Since the structure of the COSP is more abundant and comprehensive, a set of generation rules for the controller structure are put forward to exclude some unreasonable or infeasible structures. These rules can transform mixed-variable solutions into effective controllers while ensuring sufficient expression ability of solutions.

The rest of this paper is structured as follows: Section II presents the controller optimization problem; Section III presents the detailed description of the proposed controller design method; Section IV carries out simulations and experiments to validate the proposed method; and Section V concludes the whole paper.

II. PROBLEM DESCRIPTION

The controller design method proposed in this paper takes the performance indicators of the control system as the optimization objective, and designs the control law directly for the system in time domain. It can describe the dynamic characteristics of the system more completely, and has a clear design meaning and practical significance. The automatic design of a controller can be formulated as an optimization problem as follows:

\[
\min_{c} \quad J = f(p, c) \\
\text{s.t.} \quad g_i(p, c) \leq 0, \quad i = 1, 2, ..., m,
\]

where \( f(p, c) \) represents the functional relationship among the objective function value \( J \), the control object \( p \), and the controller \( c \). The control object \( p \) is usually depicted by linear or nonlinear dynamic equations. The objective function
This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TIE.2021.3114700, IEEE Transactions on Industrial Electronics

A. Controller Components

The selection of controller components plays an irreplaceable role in the controller design and should be designed properly before the application of the proposed method to ensure the quality of the obtained controllers. The controller component refers to the basic units of a controller and rich components contribute to the variety of controllers. However, the number of components will directly determine the dimension of the solution, which will affect the complexity of the solving process. Therefore, the number of components should be determined to maintain a trade-off between the diversity of structure and computational complexity. Besides, the selection of components is critical to control performance. Thus, the number and types of components should be determined first.

### III. PROPOSED CONTROLLER DESIGN METHOD

#### A. Controller Components

As shown in Table II, $A_{i,IN}$ and $A_{k,OUT}$ represent the input and output of the $i$th component, respectively. The horizontal head of the table is the Input $x$, the output of each component $A_{i,OUT}$ and the system output $y$. The vertical head of the

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**TABLE I**

**SOME COMMONLY USED ATOMIC AND COMPOSITE COMPONENTS.**

<table>
<thead>
<tr>
<th>Components</th>
<th>Proportional</th>
<th>Integral</th>
<th>Differential</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transfer function</td>
<td>$b_0$</td>
<td>$b_1/s$</td>
<td>$k_0t$</td>
</tr>
<tr>
<td>Components</td>
<td>Time-delay</td>
<td>Double integral</td>
<td>Double differential</td>
</tr>
<tr>
<td>Transfer function</td>
<td>$e^{-sT}$</td>
<td>$s^2$</td>
<td>$s^{-2}$</td>
</tr>
<tr>
<td>Components</td>
<td>First-order inertial element</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
</tr>
<tr>
<td>Transfer function</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
<td>( \ldots )</td>
</tr>
</tbody>
</table>
The input of each component $A_{k,N}$ and the controller output $u$. Each element in the table represents the connection relationship of the two corresponding ports. Specifically, value 1 represents a positive connection, -1 indicates a negative connection, and 0 stands for no connection. The connection relationship of these ports can be determined row by row. There are totally $k^2 + 3k + 2$ effective elements. So that, once the values of these elements are fixed, the controller structure will be determined. That is, the structure of the controller can be represented by a discrete vector of $k^2 + 3k + 2$ dimensions. Hence, after the parameterization of the structure, a controller solution can be represented as follows:

$$X = \left\{ X_1, X_2, \ldots, X_{d1}, X_{d1+1}, X_{d1+2}, \ldots, X_{d1+d2} \right\}$$

where $d1$ is the dimension of the parameter variable, namely the total number of the parameters of the $k$ components; $d2$ is the dimension of the structure variable and $d2 = k^2 + 3k + 2$.

As a simple example, the structure of the controller shown in Fig. 1(c) can be presented by a table as shown in Table III. Moreover, the structure solution can be represented by a 16-dimensional discrete vector $X_{struc} = [1, 1, 0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 1, 1, -1, -1, 0, 0]$.

### C. Generation Rules for Controller Structure

As mentioned above, the structure variable of the controller is a discrete vector, and each element has three values of 0/1/-1. When the length of the vector is $h$, the size of the structure solution space is $3^h$. It is apparent that the size of the solution space will increase exponentially with $h$, which will be extremely large. Besides, in this huge structure solution space, there may not be a connected channel between the input and output of the controller or not all the connections are significant. Thus, there are a large number of infeasible or unreasonable structures, and it is necessary to design a set of generation rules to remove these unnecessary structures and improve the solving efficiency. The generation rules cannot only compress the structure solution space, but also ensure the feasibility of generated structures.

### TABLE III

<table>
<thead>
<tr>
<th>Port</th>
<th>$A_{k,N}$</th>
<th>$B_{k,N}$</th>
<th>$A_{k,1/N}$</th>
<th>Output $u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$A_{k,OUT}$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$B_{k,OUT}$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$C_{OUT}$</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

As mentioned above, the structure variable of the controller is a discrete vector, and each element has three values of 0/1/-1. When the length of the vector is $h$, the size of the structure solution space is $3^h$. It is apparent that the size of the solution space will increase exponentially with $h$, which will be extremely large. Besides, in this huge structure solution space, there may not be a connected channel between the input and output of the controller or not all the connections are significant. Thus, there are a large number of infeasible or unreasonable structures, and it is necessary to design a set of generation rules to remove these unnecessary structures and improve the solving efficiency. The generation rules cannot only compress the structure solution space, but also ensure the feasibility of generated structures.

### TABLE IV

**NOTATION DESCRIPTION**

$S_{Com}$: Component state vector. If a component is connected, $S_{Com}(i) = 1$; otherwise, $S_{Com}(i) = 0, i = 1, 2, \ldots, N_{Com}$.

$S_{Edg}$: Edge state vector. If there is a connection edge between certain two ports, $S_{Edg}(j) = 1$; otherwise, $S_{Edg}(j) = 0, j = 1, 2, \ldots, N_{Edg}$.

$S_{Edp}$: Export port state vector. If it is an available export port, $S_{Edp}(k) = 1$; otherwise, $S_{Edp}(k) = 0, k = 1, 2, \ldots, N_{Com}$.

$S_{Inp}$: Input port state vector. If it is an available input port, $S_{Inp}(k) = 1$; otherwise, $S_{Inp}(k) = 0, k = 1, 2, \ldots, N_{Com}$.

$S_{Cur}$: Current connected components vector. If a component is connected in the current channel, $S_{Cur}(i) = 1$; otherwise, $S_{Cur}(i) = 0$.

In the proposed generation rules for controller structure, control channels of a controller are divided into the main channel and several branch channels. The main channel is the signal path between the input and output ports of a controller and branch channels refer to the signal loops within the controller. The following rules are adopted to construct the structure of a controller: (1) The main channel will be constructed first, then the branch channels; (2) The number of components will be constructed first, then the types of components; (3) Determine the components first and then the connection edges.

### TABLE V

**STEPS OF THE CONTROLLER STRUCTURE GENERATION.**

1. Initialize the related state vectors. Set the number of the remaining edges as $N_{Edg} = N_{Edge}$.
2. Determine the number of components in the main channel ($N_{Com}$) by roulette approach. Then select $N_{Com}$ components randomly from the given components and connect them with the input and output ports end to end. Update the related state vectors. $N_{Edg} = N_{Edg} - N_{Com} - 1$.
3. If $N_{Edg} > 0$, then select an available export port randomly as the start of the current edge. If $N_{Edg} > 1$, go to Step 4. Otherwise, skip to Step 7.
4. Generate a random real number $(\alpha)$ in $[0, 1]$. If $\alpha < 0.5$, skip to Step 5. Otherwise, skip to Step 6.
5. Select an unconnected component randomly to build a branch channel, then go to Step 7. $N_{Edg} = N_{Edg} - 1$.
6. Select an available input port randomly as the end of the current edge. Determine the value of the connected edge according to the connection probability matrix and update the related state vectors. $N_{Edg} = N_{Edg} - 1$.
7. If $N_{Edg} > 1$, then return to Step 4. If $N_{Edg} = 1$, then return to Step 6. If $N_{Edg} = 0$, then stop.

Once the number of components ($N_{Com}$) is determined, the total number of connection edges ($N_{Edge}$) of the controller can be determined as aforementioned and $N_{Edge} = d2$. During the construction of the controller structure, some state vectors are used to record the state of each component and each edge as listed in Table IV. With the given components, the structure of a controller can be determined as listed in Table V.

### D. Process of the COSP Design

Before the implementation of this method, the number and types of components should be determined based on the property of the control object, considering both the structure diversity and computational complexity. Then, the structure and parameters of a controller are evolved synchronously.
by intelligent algorithms, while viewing the structure and parameters as an organic whole during the evaluation process of solutions (as shown in Fig. 2). Firstly, the population will be initialized randomly. Then, JADE (mainly including mutation and crossover operators) will be carried out to evolve the parameter part of solutions and EDA will be executed to update the structure part of solutions iteratively using the generation rules proposed in Sec. III. C. Then, the better half of the parent and offspring solutions will be propagated into the next generation. Furthermore, the proposed method will rebuild the population when the genetic diversity of the current population reduces to a certain level. The population will be evolved iteratively until the termination criterion is met.

IV. SIMULATION AND EXPERIMENT RESULTS

In order to validate the effectiveness of the proposed design method, the method is applied to a magnetic levitation system (MLS), which is a typical nonlinear system, with external disturbance. The structure diagram of MLS is shown in Fig. 3. Moreover, both simulations and physical experiments of the MLS are carried out in this paper. Since the parameters of the actual plants vary from one to another, controllers designed by the proposed method are compared with advanced control algorithms only in the simulation.

![Fig. 3. The structure diagram of the MLS.](image)

A. Mathematical Model of the Controlled Object

The magnetic levitation system (MLS) is a kind of single-degree-of-freedom control system. As a nonlinear system, it is a typical object to verify various control algorithms, especially nonlinear control algorithms [28], [29]. In order to control a magnetic levitation system, it is usually demanded to get a linear model by linearizing the model near the equilibrium point of the system, and then a wide variety of controllers have been designed [27], [30], [31]. Hence, the method proposed in this paper is applied to MLS to verify its performance. The following dynamic model of MLS is considered in this paper:

\[
\begin{aligned}
\frac{d^2 x(t)}{dt^2} &= F(i, x) + mg \\
mg + F(i_0, x_0) &= 0 \\
F(i, x) &= K\left(\frac{x}{x_0}\right)^2 
\end{aligned}
\]

where \( K = -\mu_0 K_f A N^2 \), \( x \) is the air gap between the center of mass of the ball and the magnetic pole of the electromagnet, \( m \) is the mass of the ball, \( F(i, x) \) is the electromagnetic force, \( \mu_0 \) is magnetic permeability of vacuum, \( A \) is the cross-sectional area of the magnetic circuit, and \( N \) is the number of the electromagnet coils. Besides, \( i_0 \) and \( x_0 \) are the coil current and the air gap at the time when the ball is in equilibrium.

B. Setting of the Controller Design Method

For a successful application of the proposed design method to MLS, some key elements in this method are determined for the controlled object as follows:

1) Component Selection: In the controller design for MLS, the CRTC [27] component \( (A_1) \), the proportion component \( (A_2) \), the integral component \( (A_3) \), and the differential component \( (A_4) \) are selected to constitute the controller. The CRTC component is mainly used to enhance the robust performance of the control system. Among them, the input of the CRTC component is fixed, and its output is directly connected to the output of the controller. Besides, the input of the controller includes input signal \( x \) and system output as feedback signal \( f \). So a solution of this controller optimization problem has \( 7 \) controller parameter elements and \( 24 \) structure elements. Use \( X_1, X_2, \ldots, X_7 \) to denote the proportion, integral, differential components and the CRTC parameters respectively, and use \( X_8, X_9, \ldots, X_{31} \) to represent structure variable.

2) Selection of the Optimization Objective: In order to validate the practicability and effectiveness of the proposed method, two different kinds of optimization objectives are designed to observe that whether the obtained COSPs can meet various design goals as follows:

a. ITAE: \( J = \int_0^{\infty} t \left| e(t) \right| dt \);

b. the IAE and overshoot indicators of the control system output are used as constraints, and the rise time indicator is taken as the optimization objective.

3) Optimizer Selection: Two powerful intelligent methods, namely, the estimation of distribution (EDA) algorithm [32] and JADE [33], are employed to optimize the structure and parameter solutions, respectively. The population size is set as \( 100 \) and the maximum number of iterations is set as \( 800 \) for each algorithm. Besides, some specific parameters are set as listed in Table VI.

![Table VI. Parameter settings of Intelligent Algorithms.](image)
(PBIL) algorithm [34] has been proved to be highly effective on plenty of stationary benchmark and real-world problems. Considering that the controller structure variable is a discrete vector, PBIL is used to optimize the structure of the controller.

Remark 2: These two algorithms are taken as an example of the optimizer selection, and they are used to validate the effectiveness of the proposed controller design method. Other optimization algorithms can be used to replace them as needed. Due to limited space, we just present some brief introductions to these algorithms and the parameter setting. More details of these algorithms are available in [32], [33].

4) Constraint Setting: According to the requirements on the input and the output constraints of the actual system, set the limiting modules “Saturation1” of 0 ~ 10V and “Saturation2” of -10 ~ 0V, as shown in Fig. 5.

C. Simulation of the Magnetic Levitation System

To validate the effectiveness of the proposed controller design method, the following simulations are presented in 3 different cases. After the setting of components, objectives, and optimizers, the proposed controller design method is conducted with MATLAB/Simulink to generate the COSP controllers with satisfactory indicators automatically. In MATLAB/Simulink, the controller model for application to MLS is built as shown in Fig. 4. As shown this figure, 31 elements (i.e., k1, k2, . . . , K31) need to be optimized to get a controller with satisfactory expected performance indicators. The algorithm will run multiple times independently for each design goal and the results will be statistically analyzed. As space is limited, we only present some typical results here, more detailed results are shown in the supplemental material.

In this section, some performance indicators are adopted to evaluate the tracking performance of controllers. Specifically, t1 indicates the rise time, ts indicates the settling time, overshoot indicates the overshoot, ess indicates the steady state error, ess1 indicates the mean steady state error of the entire signal, ess2 indicates the average steady state error of second half in each step period of the square wave signal, IAE indicates the integral of the absolute value of the error with time, and ITAE indicates the integral of the product of the absolute value of the error and the time.

1) Case 1: To verify the validity of the novel generalized encoding/decoding scheme, a step-reference signal of -7V to -5V is taken as the reference signal (see Eq.(5)) and the design method proposed in [25] is taken as a competitor in this section. Besides, the 2-DOF-PID control [35] is also involved in the comparison.

\[
y(t) = \begin{cases} 
-5V, & t \geq 10; \\
-7V, & t < 10.
\end{cases} 
\]  

The simulation diagram of the control system is shown in Fig. 5 and the nonlinear model of MLS is shown in Fig. 6. The considered system model parameters are given in Table VII. The simulation settings for all these methods are set the same as that in [25] for a fair comparison, namely, the Runge-Kutta method is utilized and the sampling time is 3ms.

The simulation and statistical results are presented in Table VIII where these COSPs are designed taking ITAE as the optimization objective. The output curves of COSP1 and the competitors are given in Fig.7. As shown in Table VIII, all the indicators of the COSPs are better than those of 2-DOF-PID and the controller designed in [25], except for overshoot and ess. Nevertheless, more than 86.7% of the indicators of the obtained COSPs are better than those of 2-DOF-PID. Besides, more than 83.7% of the indicators are better than those of the controller designed in [25]. It can also be found through simulations that COSP1 has better tracking performance than the competitors, which validates the effectiveness of the proposed encoding/decoding scheme.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>25g</td>
<td>r0</td>
<td>0.1200</td>
</tr>
<tr>
<td>d</td>
<td>22mm</td>
<td>d0</td>
<td>0.0750</td>
</tr>
<tr>
<td>N</td>
<td>13.85</td>
<td>E</td>
<td>0.0034</td>
</tr>
<tr>
<td>h0</td>
<td>2450</td>
<td>K</td>
<td>0.2040</td>
</tr>
<tr>
<td>h1</td>
<td>0.6105A</td>
<td>Kt</td>
<td>0.4609</td>
</tr>
</tbody>
</table>

**TABLE VIII**

**Statistical results of performance indicators for CASE 1.**

<table>
<thead>
<tr>
<th>controller</th>
<th>t1/h</th>
<th>t2/h</th>
<th>overshoot</th>
<th>ess</th>
<th>IAE</th>
<th>ITAE</th>
<th>Tc/h</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-DOF-PID [35]</td>
<td>1.7910</td>
<td>2.2656</td>
<td>0.00000</td>
<td>0.0000</td>
<td>7.3452</td>
<td>23.3659</td>
<td>N.A.†</td>
</tr>
<tr>
<td>SOC [25]</td>
<td>0.1200</td>
<td>0.2220</td>
<td>0.0637</td>
<td>0.0001</td>
<td>0.5548</td>
<td>1.6578</td>
<td>4.4019</td>
</tr>
<tr>
<td>COSP1</td>
<td>0.0480</td>
<td>0.1200</td>
<td>0.2193</td>
<td>0.0003</td>
<td>0.3601</td>
<td>0.5848</td>
<td>1.319</td>
</tr>
<tr>
<td>COSP2</td>
<td>0.1170</td>
<td>0.2040</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.3243</td>
<td>1.2720</td>
<td>1.312</td>
</tr>
<tr>
<td>COSP3</td>
<td>0.0510</td>
<td>0.0510</td>
<td>0.3925</td>
<td>0.0000</td>
<td>0.2762</td>
<td>0.4793</td>
<td>1.364</td>
</tr>
<tr>
<td>COSP4</td>
<td>0.0650</td>
<td>0.0750</td>
<td>0.0000</td>
<td>0.0034</td>
<td>0.4855</td>
<td>2.9470</td>
<td>1.315</td>
</tr>
<tr>
<td>COSP5</td>
<td>0.0510</td>
<td>0.0510</td>
<td>0.7084</td>
<td>0.0000</td>
<td>0.2553</td>
<td>4.4019</td>
<td>1.316</td>
</tr>
</tbody>
</table>

† Since the 2-DOF-PID control method was designed through theoretical analysis based on the plant characteristics, the time cost to obtain the controller is not available.

**Fig. 5.** The simulation diagram of the control system for CASE 1.

**Fig. 6.** The nonlinear model of MLS.

**Fig. 7.** The system output curves for CASE 1.
2) Case 2: To validate the superiority of the method in anti-interference performance, an existing state of art control, i.e., the adaptive linear ADRC method [18], is adopted as a competitor. A step signal from 0mm to 10mm is input as the reference signal (see Eq. (6)) and a 20N external interference force will be applied to the plant at 2s.

\[
y_{r2}(t) = \begin{cases} 
10 \text{mm}, & t \geq 0; \\
0 \text{mm}, & t < 0;
\end{cases} \quad (6)
\]

The simulation diagram of the control system is shown in Fig.8. The control plant is implemented by Matlab S-function. All the model parameters and simulation settings are set as the same with those in [18] and the sampling time is set as 1ms.

3) Case 3: In this section, COSPs are designed by intelligent algorithms for an MLS with an external acceleration disturbance \(d(t) = 2\sin(4t) \text{ m/s}^2\). In the simulation, the parameters of the MLS, the desired input signal, and the initial conditions are set as the same as those in [27] \((m = 0.54kg, x_\infty = 0.007987m, Q = 0.001624\text{H-m})\) for a fair comparison. The initial position of the ball is set as 0.02m, and the initial velocity is set as 0.01m/s. As what [27] did, the reference signal is generated by filtering a rectangular wave with a period of 4s through a third-order filter \(G_f(s)\) depicted as

\[
\frac{60 \times 70^2}{(s + 60)(s + 70)^2}. \quad (7)
\]

The rectangular wave is given as follows:

\[
y_{r3}(t) = \begin{cases} 
12 \text{mm}, & 4k \leq t \geq 4k + 4(k = 0, 2, 4 \ldots) ; \\
4 \text{mm}, & 4k \leq t \geq 4k + 4(k = 1, 3, 5 \ldots) .
\end{cases} \quad (8)
\]
The simulation diagram of the control system is similar with that of Case 1. The simulation and statistical results are shown in Table X, and Table XI corresponding to the aforementioned two optimization objectives, respectively. The bold part indicates that the index of COSP is better than that of CRTC, otherwise the part is shaded. To validate the effectiveness of the simultaneous optimization of the structure and parameters, a parallel combination of PID and CRTC (denoted by CRTC_PID) with fixed structure is considered in the comparison.

### Table X

**Statistical results of performance indicators. CRTC** refers to the CRTC with the optimized parameters.

<table>
<thead>
<tr>
<th>controller</th>
<th>t_s</th>
<th>t_0s</th>
<th>overshoot</th>
<th>ess1</th>
<th>ess2</th>
<th>IAE</th>
<th>ITAE</th>
<th>T_int</th>
</tr>
</thead>
<tbody>
<tr>
<td>COSP1</td>
<td>0.189</td>
<td>0.368</td>
<td>0.0010</td>
<td>6.68E-05</td>
<td>0.0010</td>
<td>0.0011</td>
<td>0.0011</td>
<td>0.0011</td>
</tr>
<tr>
<td>COSP2</td>
<td>0.189</td>
<td>0.368</td>
<td>0.0010</td>
<td>6.68E-05</td>
<td>0.0010</td>
<td>0.0011</td>
<td>0.0011</td>
<td>0.0011</td>
</tr>
<tr>
<td>COSP3</td>
<td>0.189</td>
<td>0.368</td>
<td>0.0010</td>
<td>6.68E-05</td>
<td>0.0010</td>
<td>0.0011</td>
<td>0.0011</td>
<td>0.0011</td>
</tr>
<tr>
<td>COSP4</td>
<td>0.189</td>
<td>0.368</td>
<td>0.0010</td>
<td>6.68E-05</td>
<td>0.0010</td>
<td>0.0011</td>
<td>0.0011</td>
<td>0.0011</td>
</tr>
<tr>
<td>COSP5</td>
<td>0.189</td>
<td>0.368</td>
<td>0.0010</td>
<td>6.68E-05</td>
<td>0.0010</td>
<td>0.0011</td>
<td>0.0011</td>
<td>0.0011</td>
</tr>
<tr>
<td>COSP6</td>
<td>0.189</td>
<td>0.368</td>
<td>0.0010</td>
<td>6.68E-05</td>
<td>0.0010</td>
<td>0.0011</td>
<td>0.0011</td>
<td>0.0011</td>
</tr>
<tr>
<td>COSP7</td>
<td>0.189</td>
<td>0.368</td>
<td>0.0010</td>
<td>6.68E-05</td>
<td>0.0010</td>
<td>0.0011</td>
<td>0.0011</td>
<td>0.0011</td>
</tr>
<tr>
<td>COSP8</td>
<td>0.189</td>
<td>0.368</td>
<td>0.0010</td>
<td>6.68E-05</td>
<td>0.0010</td>
<td>0.0011</td>
<td>0.0011</td>
<td>0.0011</td>
</tr>
<tr>
<td>CRTC1</td>
<td>0.189</td>
<td>0.368</td>
<td>0.0010</td>
<td>6.68E-05</td>
<td>0.0010</td>
<td>0.0011</td>
<td>0.0011</td>
<td>0.0011</td>
</tr>
<tr>
<td>CRTC2</td>
<td>0.189</td>
<td>0.368</td>
<td>0.0010</td>
<td>6.68E-05</td>
<td>0.0010</td>
<td>0.0011</td>
<td>0.0011</td>
<td>0.0011</td>
</tr>
</tbody>
</table>

Since CRTC was designed through theoretical analysis based on the plant characteristics, the time cost to obtain the controller is not available.

### Table XI

**Statistical results of performance indicators.**

<table>
<thead>
<tr>
<th>controller</th>
<th>t_s</th>
<th>t_0s</th>
<th>overshoot</th>
<th>ess1</th>
<th>ess2</th>
<th>IAE</th>
<th>ITAE</th>
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</tr>
</thead>
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<td>6.68E-05</td>
<td>0.0010</td>
<td>0.0011</td>
<td>0.0011</td>
<td>0.0011</td>
</tr>
<tr>
<td>COSP2</td>
<td>0.188</td>
<td>0.368</td>
<td>0.0010</td>
<td>6.68E-05</td>
<td>0.0010</td>
<td>0.0011</td>
<td>0.0011</td>
<td>0.0011</td>
</tr>
<tr>
<td>COSP3</td>
<td>0.188</td>
<td>0.368</td>
<td>0.0010</td>
<td>6.68E-05</td>
<td>0.0010</td>
<td>0.0011</td>
<td>0.0011</td>
<td>0.0011</td>
</tr>
<tr>
<td>COSP4</td>
<td>0.188</td>
<td>0.368</td>
<td>0.0010</td>
<td>6.68E-05</td>
<td>0.0010</td>
<td>0.0011</td>
<td>0.0011</td>
<td>0.0011</td>
</tr>
<tr>
<td>COSP5</td>
<td>0.188</td>
<td>0.368</td>
<td>0.0010</td>
<td>6.68E-05</td>
<td>0.0010</td>
<td>0.0011</td>
<td>0.0011</td>
<td>0.0011</td>
</tr>
<tr>
<td>COSP6</td>
<td>0.188</td>
<td>0.368</td>
<td>0.0010</td>
<td>6.68E-05</td>
<td>0.0010</td>
<td>0.0011</td>
<td>0.0011</td>
<td>0.0011</td>
</tr>
<tr>
<td>COSP7</td>
<td>0.188</td>
<td>0.368</td>
<td>0.0010</td>
<td>6.68E-05</td>
<td>0.0010</td>
<td>0.0011</td>
<td>0.0011</td>
<td>0.0011</td>
</tr>
<tr>
<td>COSP8</td>
<td>0.188</td>
<td>0.368</td>
<td>0.0010</td>
<td>6.68E-05</td>
<td>0.0010</td>
<td>0.0011</td>
<td>0.0011</td>
<td>0.0011</td>
</tr>
<tr>
<td>CRTC1</td>
<td>0.188</td>
<td>0.368</td>
<td>0.0010</td>
<td>6.68E-05</td>
<td>0.0010</td>
<td>0.0011</td>
<td>0.0011</td>
<td>0.0011</td>
</tr>
<tr>
<td>CRTC2</td>
<td>0.188</td>
<td>0.368</td>
<td>0.0010</td>
<td>6.68E-05</td>
<td>0.0010</td>
<td>0.0011</td>
<td>0.0011</td>
<td>0.0011</td>
</tr>
</tbody>
</table>

Since CRTC was designed through theoretical analysis based on the plant characteristics, the time cost to obtain the controller is not available.

Fig. 10. The system output of COSP17 compared with CRTC. CRTC1 represents the original CRTC [27], while CRTC2 indicates the CRTC with the optimized parameters obtained using JADE.

It can be found from Table X that all the IAE, ITAE, t_s, overshoot, ess2 indicators of the COSPs are better than those of CRTC. In addition to t_s, only 6 COSPs have slightly worse ess1 and IAE. More than 71.14% of all the evaluation indicators of the obtained COSPs are superior to those of CRTC. Most indicators of COSP17 outperform those of CRTC_PID, which shows the advantage of simultaneous optimization of structures and parameters. The output curves of COSP17 and CRTC are given in Fig. 10. Note that the yellow dashed line indicates the CRTC with the optimized parameters obtained using JADE (denoted by CRTC2). The performance of CRTC has been improved remarkably, except for t_s. Since ITAE is a comprehensive indicator, which is calculated by time integration, it is easy to lead to a preference for the dynamic tracking characteristics of the system response, and ignoring response speed in a certain degree. This is a normal result and phenomenon.

For better performance of COSPs, the objective function with constraints is adopted, in which IAE and overshoot are used as the constraint and the rise time is used as the design goal. It is expected that controller solutions with both excellent comprehensive performance and dynamic indicators can be obtained. The statistical results with this objective function are shown in Table XI. As shown in Table XI, the t_s indicator of the solution has been improved while other indicators become slightly worse. Because there are contradictions among these indicators, when the rise time indicators become better, it will affect the real-time tracking performance of the system and cause some sacrifices of the overshoot indicator. In summary, 92.1% of the performance indicators of COSPs are superior to those of CRTC, which shows the effectiveness of the design of objective function.

### Fig. 11

Fig. 11. The parameter sensitivity analysis of the plant parameters.

In general, when the design goal is determined, the method can stably get effective controllers by optimization with the objective as the search direction, as shown in these simulations. Thus, the setting of the objective function will directly affect the design results. As long as the objective function is set, the algorithms can automatically generate controller solutions that meet the design objectives well.

### D. Experiments of the Magnetic Levitation System

To further validate the performance of the proposed controller design method, physical experiments are implemented.

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on a laboratory-scale MLS manufactured by Googol Technology Ltd as depicted in Fig. 12. The experiment setup consists of the mechanical unit (an electromagnet, a metallic ball, and a LED-optoelectronic sensor) and the control interface.

In this setup, the feedback signal from the LED-optoelectronic sensor to the computer and control signals from the computer to the physical system are sent via PCI 1711 A/D card made by Advantech Technologies. The real-time control algorithm in the experiment is implemented by Matlab/simulink software, which contains Real-Time Workshop (RTW) tool set. The real-time experiment environment of the MLS testbed is presented in Fig. 13. The parameters of the actual MLS are the same as listed in Table VII. The environment of the practical control system of MLS is set as follows.

2) Algorithm settings: fixed step, ode4 (Runge-Kutta), and the sampling time is 0.003s.
3) Run time: 0 seconds start, manual stop.

Fig. 12. MLS testbed from Googol Technology Ltd.

Fig. 13. Experiment environment of the MLS testbed.

First, COSPs are designed by the proposed method for the actual control plant taking IAE as optimization objective. Then, apply the COSP obtained to the experimental platform where the ball is expected to move in a range of [0.01m, 0.014m] and the given input signal is a rectangular wave with a period of 9s. An obtained COSP is taken as an example and applied to the actual plant. The selected solution is

\[ X = [0.002, 0, 0.0245, 27.7534, 33.6203, 1.1714, 25.8047, 1.0, 1.0, -1.0, 0, -1.0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1.0, -1.0] \]

Thus, the corresponding simulation model built in MATLAB/Simulink for this solution is presented in Fig. 14.

Fig. 14. The COSP controller applied in the MLS testbed.

Fig. 15 demonstrates the distinction of the response curves in the simulation and experiment. More specifically, in the actual control system, \( t_r = 0.0045s \), \( t_s = 1.5780s \), \( e_{ss} = 1.9692E - 05 \), \( IAE = 4.5345E - 03 \), and \( ITAE = 7.1301E - 02 \). Most of the indicators are of the same order as the simulation system. Comparing the simulation and experiment output results within 0 to 36s, the mean absolute error of them can be calculated as shown in the following formula.

\[ Err_{mean} = \frac{1}{n} \sum_{i=1}^{n} |Y_{i,\text{sim}} - Y_{i,\text{real}}| \approx 1.6507E - 04 \]

where \( n \) is the sample size, \( Y_{i,\text{sim}} \) is the simulation output data, and \( Y_{i,\text{real}} \) is the experiment output data.

Fig. 15. The output of the simulation system and the actual system.

As can be seen in Fig. 15, the output curve of the actual system is basically the same with that of the simulation system. The main difference between the simulation output and experiment output is that some minor jitter can be found in the experiment output and the overshoot of the experiment output is larger. The reason behind this phenomenon might be that there are some internal and external disturbances in the actual system. Note that the controller by simulation optimization proposed in this paper can be directly applied to the actual control system without adjustment, which verifies the feasibility and practicability of the proposed method. Besides, when the disturbance is imposed on the object, the feasibility and practicability of the proposed method. Moreover, the proposed design method is applied to a typical

V. Conclusion

In this paper, controllers are designed automatically in an evolutionary manner under a generalized structure encoding/decoding scheme. The design of controllers is formulated as a mixed-variable optimization problem. A generalized encoding and decoding scheme is proposed to represent a controller. The structure and parameters of a controller are optimized simultaneously under the framework of evolution. Besides, a set of generation rules for the controller structure is proposed to remove some unreasonable and infeasible structures, which can also ensure the feasibility of solutions. Moreover, the proposed design method is applied to a typical
nonlinear system, namely a magnetic levitation system (MLS) with external disturbance. The validity of the proposed controller design method has been proved by the simulations and experiments about MLS. Comparing the numerical results with those of the state-of-the-art controller methods, the performance indicators have been remarkably improved, which proves the practicability and effectiveness of the proposed design method. The proposed method can not only shorten the design cycle, but can also reduce the dependence of controller design on human beings.

The proposed method can easily find a controller with desirable performance through automatic design. Besides, the introduction of the knowledge of control objects and the design experience in the selection of components is conducive to improving the solving efficiency of the proposed method. The number of components should be moderate so as to find controllers with satisfactory expected indicators within the acceptable computing time. The proposed method relies on the design experience of human beings to some extent. The dependence mainly lies in the selection of candidate components to constitute a controller. To further reduce the dependence on human beings, automatic selection of candidate components deserves further research in the future. It is worth noting that the choice of intelligent optimization algorithms is not the focus of this paper. The EDA and JADE algorithms used in the simulation and experiment are taken as a choice with good performance after a comparison. Undoubtedly, they can be replaced by other competent algorithms as needed. In addition, more applications to diverse control objects will be considered in future work.

ACKNOWLEDGMENT

The authors would like to thank Prof. Kuangang Fan for providing the source code of A-LADRC for comparison.

REFERENCES

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