# A Multi-objective Comprehensive Learning Particle Swarm Optimization with a Binary Search-Based Representation Scheme for Bed Allocation Problem in General Hospital

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***Abstract*—Bed allocation is a crucial issue in hospital management. This paper proposes a multi-objective comprehensive learning particle swarm optimization with a representation scheme based on binary search (BS-MOCLPSO) to deal with this problem in general hospital. The bed allocation problem (BAP) is first modeled as an M/PH/c queue. Based on the queuing theory, the mathematical forms of admission rates and bed occupancy rate is deduced for each department of the hospital. Taking the maximization of both rates as objectives, the BS-MOCLPSO generates a set of non-dominated optimal allocation decisions for the hospital manager to select. The proposed algorithm introduces a novel binary search-based representation scheme, which transforms a particle's position into a feasible allocation scheme through binary search. Simulation results on real hospital data show that the proposed algorithm can offer allocation decisions that lead to higher service level and better resource utilization.**

***Keywords*—particle swarm optimization, multi-objective particle swarm optimization, binary search, bed allocation**

## I. INTRODUCTION

Hospital bed is an important resource of the medical facility. An under-provision of hospital beds leads to low admission rates of patients who need hospital care, whereas an over-provision of hospital beds leads to a waste of resources. A general hospital is composed of a number of departments to serve patients with different diseases. The bed allocation among these departments is restricted by the limited recourses. That is, the total number of beds in the hospital is fixed. In many hospitals, it is very common that beds in some departments are overload running, whereas some beds in other departments are left unused. Allocating the beds in these departments is thus important and promising for improving the service level and the resource usage of the whole hospital.

In recent years, the bed allocation problem (BAP) has received increasing attentions. Kinds of mathematical models for BAP were built in the literature, among which the uses of queuing theory are very common [1]-[7]. In [4] and [7], the model of M/PH/$c$ queues was proposed, where M denotes patient arrival subject to Poisson distribution (Markov arrivals), PH denotes patient lengths of stay (LoS) subject to Phase-type distribution, and $c$ is the number of beds. In this paper, we solve the BAP based on the M/PH/$c$ queuing model [4][7].

In the literature, some methods have been proposed to solve the BAP. However, some of them considered only one objective, e.g., maximizing the patient admission rate [8] or maximizing the bed occupancy [9]. These works cannot address the complex relationship between the two objectives. As an increment in the admission rate may result in a decrease in the occupancy rate and vice versa, an optimal bed allocation should strike a balance between the two objectives. Some works concerned only one department at a time [10], ignoring the constraint of the limited recourses in the hospital. Some works were designed for a specified department [11][12][13], and were not suitable for the other departments.

The BAP is NP-completed [14] and always high-dimensional. Recent years, evolutionary algorithms (EAs) have been widely used to deal with these complex problems [15]-[19]. And in this paper, we propose a multi-objective comprehensive learning particle swarm optimization with a novel binary search-based particle representation scheme (BS-MOCLPSO) to solve the BAP in general hospital. The objectives of maximizing the patient admission rate and maximizing the bed occupancy rate are both taken into account. The BS-MOCLPSO outputs a collection of non-dominated solutions along the Pareto-optimal front. Hospital manager can select one solution from the non-dominated collection according to his preference and use it as a guideline for allocating beds to hospital departments.

Another novelty of the proposed algorithm is to introduce a binary search-based representation scheme to keep the total number of hospital beds as a fixed number. In resource allocation problems (RAPs), exploring the large search space always yields infeasible solutions violating the pre-specified fixed resource bounds [20][21]. Using penalty strategies [22] to "publish" the infeasible solutions is a way to solve the RAPs, but it cannot guarantee the generation of a feasible solution. We can also use the repair strategies to "repair" the infeasible solutions. However, the population of the algorithm may oscillate due to the large number of repaired solutions. In this paper, the proposed algorithm employs a representation scheme based on a modified binary search method. Each particle can be decoded into a feasible bed allocation decision through the binary search. In this way, the algorithm can address the RAPs, avoiding the problems of penalty strategies and repaired strategies simultaneously.

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The rest of this paper is organized as follows. Section II introduces the queuing model of bed allocation. Section III proposes and describes the BS-MOCLPSO. Experiments are conducted in section IV, in which the case study comes from the 323-general hospital of PLA, China. Finally, conclusions are drawn in Section V.

## II. MATHEMATICAL MODEL OF BED ALLOCATION

According to [4][7], the model of BAP is defined as a M/PH/$c$ queue. Suppose that the hospital has $n$ beds and $N$ departments. For each department $i$, $i$=1,2,…,$N$, the number of beds is $c^i$, the Poisson arrival rate is $\lambda^i$ and the mean LoS calculated by the Phase-type distribution is $\tau^i$. We deduce that the mean number of arrivals during a LoS is $\lambda^i\tau^i$. Thus, according to Erlang's loss formula [23], the probability of $c^i$ beds are all occupied is

$$p^i = [(\lambda^i\tau^i)^{c^i}/c^i!] \Big/ \sum_{k=0}^{c^i}[(\lambda^i\tau^i)^k/k!] \ . \qquad (1)$$

Therefore, we deduce the admission rate of patient in department $i$ as

$$p_a^i = 1 - p^i \ . \qquad (2)$$

The mean number of occupied beds or the carried load in department $i$ is

$$l^i = \lambda^i\tau^i(1-p^i) \ . \qquad (3)$$

Therefore, the bed occupancy can be defined as

$$p_o^i = l^i/c^i = \lambda^i\tau^i(1-p^i)/c^i \ . \qquad (4)$$

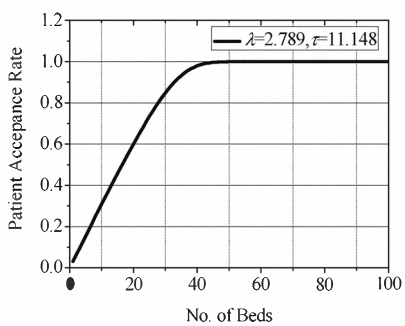


Figure 1. Relationship between admission rate and number of beds.

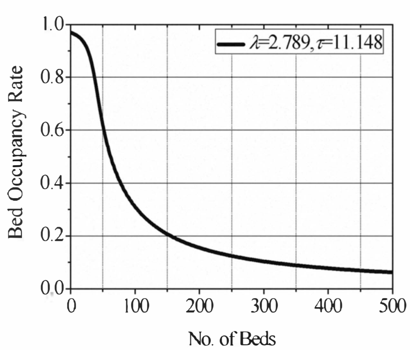


Figure 2. Relationship between bed occupancy and number of beds.

According to (2) and (4), suppose that $\lambda^i$ =2.789 and $\tau^i$ =11.148, the admission rate and the bed occupancy for different numbers of beds $c_i$ is shown in Fig. 1 and Fig. 2, respectively. We can see that, when the number of beds is small (less than 20), the admission rate is awfully low, whereas the bed occupancy is high and up to 100%. With the increase of the number of beds, the admission rate increases to 100%, whereas the bed occupancy decreases to almost zero. It can be observed that an allocation of over fifty beds to the department causes heavy waste of resources.

## III. MOCLPSO WITH BINARY SEARCH-BASED REPRESENTATION SCHEME

In this paper, a multi-objective comprehensive learning PSO (MOCLPSO) [24] is proposed to solve the bed allocation problem. We also employ a novel binary search-based representation scheme to the MOCLPSO (BS-MOCLPSO), by which the dimensionality of the search space is twice as the number of departments in the hospital.

### A. *Binary Search-Based Representation Scheme*

The binary search-based representation scheme [25] was first proposed for the ant colony-inspired (ACI) algorithm to deal with constraints in continuous problem space. In this paper, we modify the scheme and apply it to deal with BAP, which is a kind of RAPs and belongs to discrete combinatorial optimization problems (COPs).

In BS-MOCLPSO, the position of particle $i$ is represented by

$$\boldsymbol{X}_i = (\boldsymbol{x}_i, \boldsymbol{y}_i) = (x_i^1, x_i^2, ..., x_i^N, y_i^1, y_i^2, ..., y_i^N), \qquad (5)$$

where $\boldsymbol{x}_i = (x_i^1, x_i^2, ..., x_i^N)$ and $\boldsymbol{y}_i = (y_i^1, y_i^2, ..., y_i^N)$ are floating-point vectors subject to (6) and (7) respectively.

$$g(\lfloor \boldsymbol{x}_i \rfloor) = g(\lfloor x_i^1 \rfloor, \lfloor x_i^2 \rfloor, ..., \lfloor x_i^N \rfloor) \le 0 \qquad (6)$$

$$g(\lfloor \boldsymbol{y}_i \rfloor) = g(\lfloor y_i^1 \rfloor, \lfloor y_i^2 \rfloor, ..., \lfloor y_i^N \rfloor) > 0 \qquad (7)$$

$$g(\boldsymbol{x}) = g(x^1, x^2, ..., x^N) = x^1 + x^2 + ... + x^N - n \qquad (8)$$

As $n$ is the number of beds and $N$ is the number of departments, $g(\boldsymbol{x})$=0 stands for the constraint that the total number of beds in the hospital is fixed. We apply a binary search decoder to decode the pair of points ($\boldsymbol{x}_i$,$\boldsymbol{y}_i$) in order to get a feasible solution $\boldsymbol{c}_i = (c_i^1, c_i^2, ..., c_i^N)$ , which is an integer vector satisfies $g(\boldsymbol{c}_i)$=0.

The pseudo code for the binary search decoder is shown in Fig. 3. In each iteration of the decoder, the midpoint of $\boldsymbol{x}_i$ and $\boldsymbol{y}_i$ is calculated. If the midpoint $\boldsymbol{p}$ satisfies $g(\lfloor \boldsymbol{p} \rfloor) = 0$, we stop the iteration and set $\boldsymbol{c}_i = \lfloor \boldsymbol{p} \rfloor$. Otherwise, if $g(\lfloor \boldsymbol{p} \rfloor) \le 0$, we move $\boldsymbol{x}_i$ to $\boldsymbol{p}$. Otherwise, we move $\boldsymbol{y}_i$ to $\boldsymbol{p}$. Moreover, the maximal number of iterations is defined as *max_it*=200. If no feasible point is found within this specified number of iterations, the particle is associated with zero fitness values in the later evaluation process, meaning that no feasible allocation decisions is found and the particle should be "punished".

Actually, in our experiment we find that the binary search decoder is good at generating a feasible $\boldsymbol{c}_i$ by using $\boldsymbol{x}_i$ and $\boldsymbol{y}_i$.

What should be emphasized is that in the binary search decoder $x_i$ and $y_i$ are local parameters, i.e., the position of particle $i$ never changes during the decoding process.

```
BSdecoder (Xi: real vector): integer vector
If g(⌊xi⌋)=0 then return ⌊xi⌋;          {output feasible solution}
End if
If g(⌊yi⌋)=0 then return ⌊yi⌋;          {output feasible solution}
End if
it = 0;
Repeat
    p = midpoint(xi, yi);
    If g(⌊p⌋) ≤ 0 then xi = p;
    End if
    If g(⌊p⌋) > 0 then yi = p;
    End if
    it = it +1;
Until g(⌊p⌋)=0 or it > max_it
Return ⌊p⌋;                              {output feasible solution}
```

Figure 3. Pseudo Code for the Binary Search Decoder.

## B. Objective Function

The admission rate of patient and the bed occupancy are optimized simultaneously. The objective functions are defined as

$$f_1 = \sum_{i=1}^{N} w_i \times p_a^i , \tag{9}$$

$$f_2 = \sum_{i=1}^{N} w_i \times p_o^i , \tag{10}$$

where $p_a^i$ and $p_o^i$ are the admission rate and bed occupancy in department $i$ according to (2) and (4); $w_i$ is the proportion of patients in department $i$ according to the statistical data in the hospital.

By this way, we consider all the departments in the hospital instead of one or two specified departments. The competition for resources among different departments in the hospital is taken into account and is solved in a relatively fair way.

## C. Overall Process of BS-MOCLPSO

The MOCLPSO algorithm applied in this paper is proposed in [24]. It uses an external archive to maintain a set of non-dominated solutions obtained during the training process. The *gBest* position in each iteration is randomly selected in the archive. Each particle learns from its exemplars until it ceases improving for *rg* iterations (refreshing gap). And then the exemplars assigned to each particle are updated. When updating the exemplars of a particle, $m = \lceil Pe \times 2N \rceil$ dimensions of the exemplars are randomly chosen from the *gBest*, where *Pe* is the elitism probability. Some of the remaining dimensions are randomly chosen to learn from the particle's own *pBest* within a learning probability (*Pc*). The remaining dimensions learn from other particles' *pBest* by a tournament selection procedure. More details about MOCLPSO can be found in [24]. In this paper, we just present an overall process of the BS-MOCLPSO, whose pseudo code is shown in Fig. 4.

```
BS-MOCLPSO algorithm
Initialize;
Repeat
  For i = 1:M do                    {M is the population size}
    If Pi.stop > rg then
        Randomly select a gBest from the external archive;
        Assign the exemplar of each dimension for Pi;
        Pi.stop = 0;
    End if
    Update Pi.V;                    {velocity update}
    Update Pi.X;                    {position update}
    BSdecoder(Pi.X);
    Evaluate Pi.X according to the output of the BSdecoder;
    If Pi.X dominates Pi.pBest then Pi.pBest = Pi.X;
    End if
    If Pi.pBest dominates Pi.X then Pi.stop = Pi.stop +1;
    End if
    If Pi.X and Pi.pBest are nondominated each other then
      r = random(0,1);
      If r<0.5 then Pi.pBest = Pi.X;
      Else Pi.stop = Pi.stop +1;
      End if
    End if
  End for
  Update the external archive;
Until stopping criterion is satisfied
Output the external archive;
```

Figure 4. Pseudo Code for the BS-MOCLPSO.

# IV. RESULTS AND DISCUSSIONS

Case study comes from the 323-general hospital of PLA, China. The hospital has 18 departments and 597 beds. Parameter settings of the MOCLPSO are shown in Table I which are according to [24], where *M* stands for the population size; *as* stands for the archive size; *Pc* is the learning probability; *Pe* is the elitism probability; $\omega$ is the inertia weight, *c* is the acceleration coefficient; *rg* is the refreshing gap; *currFes* stands for the current number of evaluations, and *maxFes* is the maximal number of evaluations.

TABLE I. PARAMETER SETTINGS

| | |
|---|---|
| $M$ | 50 |
| $as$ | 20 |
| $Pc$ | 0.1 |
| $Pe$ | 0.4 |
| $\omega$ | $0.9 - 0.7 * (\text{curr}Fes/\max Fes)$ |
| $c$ | 2.0 |
| $rg$ | 10 |
| max*Fes* | 200000 |

As the archive size is set as 20, 20 non-dominated solutions are generated by the BS-MOCLPSO for the hospital manager to select. Due to limitations of the paper length, the results are presented here only for the two boundary solutions. $S_1$ is the solution which maximizes the objective function $f_1$, and $S_2$ is the solution which maximizes the objective function $f_2$. We compare $S_1$ and $S_2$ with the previous allocation decision of the hospital. As shown in Table II, $c$ is the number of beds allocated for each department, $Pa$ is the admission rate, and $Po$ is the bed occupancy. Moreover, Fig. 5 shows the changes in performance of the patient admission rate in each department, while Fig. 6 shows the changes in performance of the bed occupancy in each department. The advantages of the allocation decision generated by our method are summarized as follows.

TABLE II. COMPARISONS AMONG $S_1$, $S_2$ AND THE PREVIOUS ALLOCATION DECISION.

| Dept. | Previous | | | $S_1$ | | | $S_2$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | *c* | *Pa* | *Po* | *c* | *Pa* | *Po* | *c* | *Pa* | *Po* |
| 1 | 40 | 97.94% | 76.13% | 34 | 91.87% | 84.01% | 30 | 84.83% | 87.92% |
| 2 | 35 | 80.64% | 91.15% | 44 | 94.00% | 84.52% | 43 | 92.90% | 85.48% |
| 3 | 35 | 94.84% | 80.80% | 34 | 93.61% | 82.10% | 33 | 92.22% | 83.32% |
| 4 | 25 | 99.95% | 48.96% | 13 | 83.60% | 78.75% | 24 | 99.90% | 50.98% |
| 5 | 15 | 69.77% | 88.29% | 25 | 96.39% | 73.19% | 17 | 77.23% | 86.24% |
| 6 | 30 | 87.85% | 86.01% | 31 | 89.67% | 84.97% | 31 | 89.67% | 84.97% |
| 7 | 35 | 94.23% | 81.64% | 28 | 81.98% | 88.78% | 38 | 97.16% | 77.53% |
| 8 | 35 | 71.03% | 94.04% | 51 | 94.29% | 85.68% | 50 | 93.34% | 86.51% |
| 9 | 22 | 86.76% | 83.36% | 25 | 93.13% | 78.75% | 18 | 74.87% | 87.93% |
| 10 | 30 | 98.16% | 71.08% | 25 | 92.09% | 80.03% | 24 | 90.12% | 81.58% |
| 11 | 35 | 93.11% | 83.03% | 34 | 91.70% | 84.18% | 32 | 88.43% | 86.25% |
| 12 | 35 | 94.10% | 81.82% | 31 | 87.92% | 86.30% | 30 | 86.00% | 87.23% |
| 13 | 25 | 52.06% | 95.97% | 48 | 91.45% | 87.81% | 49 | 92.56% | 87.06% |
| 14 | 25 | 90.32% | 81.90% | 26 | 92.22% | 80.40% | 26 | 92.22% | 80.40% |
| 15 | 55 | 92.61% | 88.05% | 59 | 95.92% | 85.01% | 52 | 89.40% | 89.90% |
| 16 | 40 | 96.98% | 78.64% | 34 | 89.90% | 85.76% | 36 | 92.84% | 83.64% |
| 17 | 40 | 99.74% | 65.00% | 27 | 87.90% | 84.86% | 32 | 95.80% | 78.04% |
| 18 | 40 | 99.49% | 68.24% | 28 | 87.38% | 85.63% | 32 | 94.11% | 80.70% |
| Total | 597 | 85.88% | 83.63% | 597 | 91.89% | 83.68% | 597 | 89.27% | 84.91% |

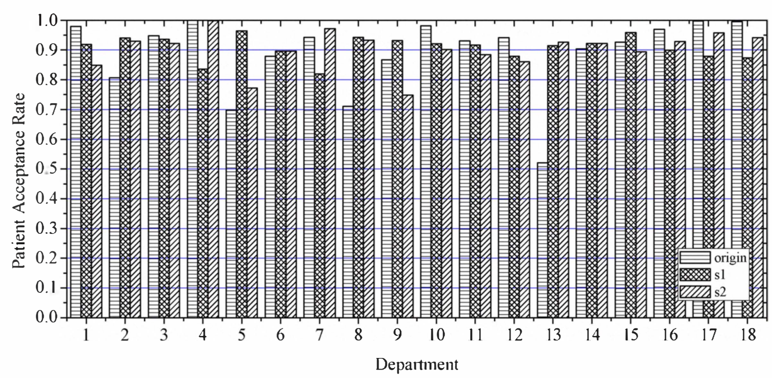


Figure 5. Comparisons of patient admission rates.

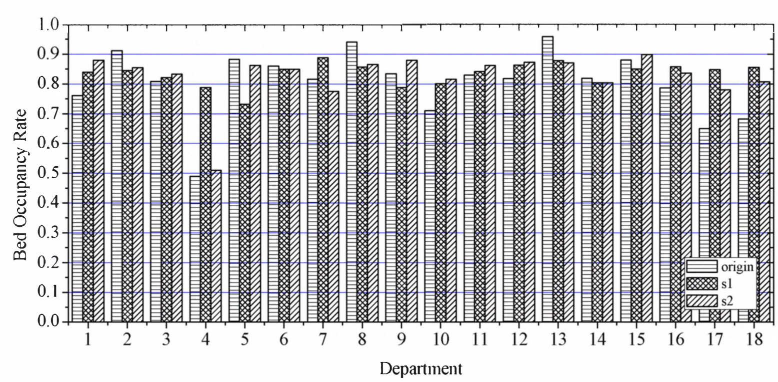


Figure 6. Comparisons of bed occupancy rates.

(1) The proposed method balances the resource competition among different departments. The working efficiency and level of management in the hospital are improved. In the previous allocation decision, department 4 has the highest admission rate about 99.95% and department 13 has the lowest admission rate about 52.06%. In contrast, in $S_1$, the highest admission rate is 96.39% in department 5 and the lowest admission rate is 81.98% in department 7. In $S_2$, the highest admission rate is 99.90% in department 4 and the lowest admission rate is 74.87% in department 9. Therefore, in the new allocation decisions, not only the admission rates are balanced, but also the lowest admission rates are remarkably improved. Similar results are also reported in bed occupancy. As can be observed, the highest and lowest bed occupancy rates are 95.97% and 48.96% in the previous situation, 88.78% and 73.19% in $S_1$, and 89.90% and 50.98% in $S_2$. Both our solutions, especially $S_1$, outperform the previous decision. We relieve the previous situation that some departments are overload running whereas some other departments are idle. Even by the practical standard that the average occupancy rate should be around 80% [7], $S_1$ offers a quite satisfying solution.

(2) From an overall point of view, the general patient admission rate and bed occupancy are increased from 85.88% and 83.63% to 91.89% and 83.68% respectively in $S_1$, to 89.27% and 84.91% respectively in $S_2$.

(3) The bed occupancy and patient admission rate are opposite objectives. We provide a set of non-dominated allocation decisions. Choice of the optimal decision is based on the preference of hospital managers.

## V. Conclusion

In this paper, we use a multi-objective comprehensive learning particle swarm optimization (MOCLPSO) to optimize the bed allocation problem (BAP). A novel representation scheme based on modified binary search is proposed to deal with the constraint that the hospital has a fixed number of resources. Optimized by the proposed BS-MOCLPSO algorithm, the beds in 323-general hospital of PLA, China is reallocated. Simulation results show that resource competition among different departments is more balanced and the general patient admission rate and bed occupancy in the hospital both increased. Thus, the service level and the resource usage in the hospital are improved simultaneously.

The proposed binary search-based representation scheme can be extended to other evolutionary algorithms (EAs). Their combinations indicate a new way to address the RAPs with resource constraints, which is also our future direction.

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