Multi-Criteria Layout Synthesis of MEMS Devices Using Memetic Computing

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***Abstract*—This paper introduces a multi-objective optimization approach for layout synthesis of MEMS components. A case study of layout synthesis of a comb-driven micro-resonator shows that the approach proposed in this paper can lead to design results accommodating two design objectives, i.e. simultaneous minimization of size and power input of a MEMS device, while investigating optimum geometrical configuration as the main concern. The major contribution of this paper is the application of memetic computing in MEMS design. An evolutionary multi-objective optimization (EMO) technique, in particular non-dominated sorting genetic algorithm (NSGA-II), has been applied to find multiple trade-off solutions followed by a gradient-based local search, i.e. sequential quadratic programming (SQP), to improve the convergence of the obtained Pareto-optimal front. In order to reduce the number of function evaluations in the local search procedure, the obtained non-dominated solutions are clustered in the objective space and consequently, a post-optimality study is manually performed to find out some common design principles among those solutions. Finally, two reasonable design choices have been offered based on manufacturability issues.**

Keywords—evolutionary muti-objective optimization; local search; knowledge discovery; MEMS design

I. INTRODUCTION

MEMS are tiny mechanical devices that are built upon semiconductor chips and are measured in micrometers. They usually integrate across different physical domains a number of functions, including fluidics, optics, mechanics and electronics, and are used to make numerous devices such as pressure sensors, gyroscopes, engines, and accelerometers etc. Many designs of MEMS are made through engineering experience and back of the envelop calculations, and are highly dependent on designers knowledge and experience. One reason for this is the complexity involved in the modeling, design and fabrication of MEMS. There are many constraints in designing and fabricating MEMS devices due to the limitations of current fabrication techniques [1, 3]. However, as process technologies become more stable, research emphasis can be shifted from developing specific process technologies towards the design of systems with a large number of reusable components, such as resonators, accelerometers, gyroscopes, and micro-mirrors. It greatly benefits the MEMS designers if the routine design of frequently used components can be optimized automatically by computer programs, while the designers can take more time in contemplating the more creative conceptual designs [1-3].

Although traditional mathematical programming oriented numerical optimization techniques have been widely used in the design optimization of MEMS, current challenging areas of optimization in general engineering design applications look for means to overcome some of the limitations within local gradient-based search by incorporating a more stochastic approach which provides essential explorative and robust search capabilities. Evolutionary Algorithms (EA) are a class of algorithms which are designed to handle complex multi-modal [4] or multi-funnel [5] design landscapes and moreover have already been incorporated into MEMS design [6-7] in which the emphasis was put mainly on planar designs [8-14, 26].

This paper presents a methodology for investigation of optimum layout synthesis of a MEMS device, i.e. a comb-driven micro-resonator, aiming at having minimum size (in other words, smallest device area), and simultaneously having minimum power (i.e. voltage), subjected to several design constraints. More specifically, the choices of different sets of geometrical design parameters for comb drive, folded flexure beam and shuttle mass (see Fig. 1) have been investigated in order to achieve the goals mentioned above which are in essence conflicting. An evolutionary multi-objective optimization (EMO) algorithm, i.e. non-dominated sorting genetic algorithm (NSGA-II) is initially performed to find the Pareto-optimal front. The non-dominated solutions found so far have been clustered based on their Euclidean distances (in the objective space) in a prefix grid structure to reduce the number of the solutions, which will in turn be served as initial starting points for the gradient-based local search technique, i.e. sequential quadratic programming (SQP). The ε-constraint method is applied by fixing the first objective (i.e. voltage) as a constraint for each clustered non-dominated solutions independently to obtain the modified optimized front. Further improvement in accuracy and confidence in the convergence of the Pareto-optimal front is achieved, and following this, a brief post-optimality study is performed to unveil some common design principles among members of the clustered Pareto-optimal set. Finally, two reasonable design solutions among those multiple trade-off solutions have been selected based on manufacturability concerns.

II. MEMS MODEL

A. Introduction to the MEMS Model

A case study in the area of MEMS design (originally taken from [23, 24]) was carried out to verify the effectiveness of the above design optimization methodology following a memetic computational approach which involves evolutionary multi-objective optimization coupled with a simple clustering algorithm as well as a gradient based local search technique. The design problem is a comb-drive micro-resonator (see the layout in Fig. 1), with fourteen mixed-type design variables (L_b, w_b, L_t, w_t, L_{sy}, w_{sy}, L_{sa}, w_{sa}, w_{cy}, L_c, w_c, x_0, V, N_c), and twenty four design constraints, both linear and nonlinear. More detailed description of the design problem in terms of analytical equations is given below.

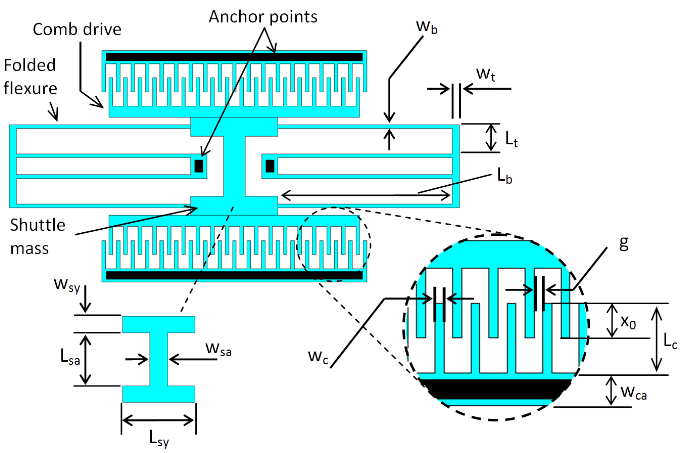

Figure 1. MEMS model (adapted from [23, 24]).

L_b and w_b are the length and the width of the flexure beam (respectively), L_t and w_t are the length and the width of the truss beam (respectively), L_{sy} and w_{sy} are the length and the width of the shuttle yoke (respectively), L_{sa} and w_{sa} are the length and the width of the shuttle axle (respectively), w_{cy} is the width of the comb yoke, L_c and w_c are the length and the width of the comb fingers (respectively), x_0 is the comb finger overlap, V is the voltage amplitude, and N_c is the number of rotor comb fingers. The equations which are necessary to build the parametric layout are given as follows, Eqns. 1-15,

$$L_1 = (N_c-1)(w_c+2g)+N_c.w_c \quad (1)$$

$$L_2 = L_1+2(g+w_c) \quad (2)$$

$$A_s = w_{sa}.L_{sa}+2w_{sy}.L_{sy} \quad (3)$$

$$A_b = 4(2L_b.w_b+(L_t+w_a/2).w_t+(w_a.w_{ba})/2) \quad (4)$$

$$A_c = 2(L_1.w_{cy}+N_c.w_c.L_c) \quad (5)$$

$$A_t = 2(w_{ca}.L_2+(N_c+1).w_c.L_c) \quad (6)$$

$$B_x = \mu((A_s+0.5A_t+0.5.A_b).(1/d+1/\gamma) + (A_c/g)) \quad (7)$$

$$\alpha = (w_t/w_b)^3 \quad (8)$$

$$K_x = (2E.t.w_b^3/L_b^3).(L_t^2+14\alpha.L_t.L_b+ 36\alpha^2L_b^2) / (4L_t^2+41\alpha.L_t.L_b+36\alpha^2.L_b^2) \quad (9)$$

$$K_y = (2E.t.w_t^3/L_t^3).(8L_t^2+ 8\alpha.L_t.L_b+ \alpha^2.L_b^2)/(4L_t^2+10\alpha.L_t.L_b+5\alpha^2.L_b^2) \quad (10)$$

$$K_{ey} = 2\varepsilon_0.N_c.V^2.x_0.t/g^3 \quad (11)$$

$$m_x = \rho(A_s+0.25A_t+(12/35).A_b).t \quad (12)$$

$$Q = (m_x.K_x/B_x^2)^{1/2} \quad (13)$$

$$F_{ex} = 1.12\varepsilon_0.N_c.(t/g).V^2 \quad (14)$$

$$x_{disp} = Q.F_{ex}/K_x \quad (15)$$

where L_1 and L_2 are the lengths of the lower and the upper comb yokes, A_s, A_b, A_c, A_t are the layout areas of the shuttle yokes, the flexure beams, the comb finger sidewalls and the truss beams, respectively, B_x is the damping coefficient, K_x and K_y are the folded flexure spring constants, F_{ex} is the lateral component of the external electrostatic force generated by the comb drives, m_x and m_y are the effective masses, Q is the quality factor and x_{disp} is the displacement amplitude. Further details about these analytical equations and derivations of them are given in [23, 24].

B. Design Criteria

Several design constrains must be defined to constrain the layout synthesis of the microresonator. There are 24 linear and nonlinear constraints defined as given in the following Eqns. 16-28.

$$g_{1,2}(\mathbf{x})= 0 < L_2 \leq 700\ (\mu m) \quad (16)$$

$$g_{3,4}(\mathbf{x})= 0 < L_{sy}+2L_b+2w_t \leq 700\ (\mu m) \quad (17)$$

$$g_{5,6}(\mathbf{x})= 0 < 2(w_{ca}+2L_c-x_0+w_{cy})+L_{sa}+2w_{sy} \leq 700\ (\mu m) \quad (18)$$

$$g_{7,8}(\mathbf{x})= 4 \leq L_c-x_0+x_{disp} \leq 200\ (\mu m) \quad (19)$$

$$g_{9,10}(\mathbf{x})= 4 \leq x_0-x_{disp} \leq 200\ (\mu m) \quad (20)$$

$$g_{11}(\mathbf{x})= (L_t+w_a/2) \leq (L_{sa}/2+w_{sy}+x_{disp})\ (\mu m) \quad (21)$$

$$g_{12}(\mathbf{x})= (L_{sa}/2+x_{disp}) \leq L_t+w_a/2-w_b\ (\mu m) \quad (22)$$

$$g_{13,14}(\mathbf{x})= 2 \leq L_{sy}/2-w_{ba}-w_{sa}/2 \leq 200\ (\mu m) \quad (23)$$

$$g_{15,16}(\mathbf{x})= 2 \leq x_{disp} \leq 100\ (\mu m) \quad (24)$$

$$g_{17,18}(\mathbf{x})= 5 \leq Q \leq 10^5 \quad (25)$$

$$g_{19,20}(\mathbf{x})= 0 \leq x_{disp}/L_b \leq 0.1 \quad (26)$$

$$g_{21,22}(\mathbf{x})= 0 < K_{ey}/K_y \leq 1/3 \quad (27)$$

$$g_{23,24}(\mathbf{x})= 4 \leq L_{sa}/2-x_{disp}-w_a/2 \leq 200\ (\mu m) \quad (28)$$

III. OPTIMIZATION METHODOLOGY

In this section, the multi-objective optimization problem (MOP), briefly described in the previous section that is related to the layout synthesis of MEMS components with respect to dynamic response (i.e. voltage) and the size of the device, is formulated. Optimum design parameters, i.e. geometrical features of the flexure beams, comb drives and the shuttle mass, are investigated to simultaneously minimize the power consumption or in other words the voltage and the area of the

MEMS device. The constrained multi-objective optimization problem is given below,

$$
\begin{aligned}
\text{Minimize } f_1(\mathbf{x})\text{:} \quad & V \\
\text{Minimize } f_2(\mathbf{x})\text{:} \quad & A_{total}=(A_s+A_t+A_b+A_c) \\
\text{subject to:} \quad & g_i(\mathbf{x}), \quad \text{for i=1, 2, …, 24} \qquad (29) \\
\mathbf{x}= \quad & \{L_b, w_b, L_t, w_t, L_{sy}, w_{sy}, w_{sa}, \ldots \\
& w_{cy}, L_c, w_c, L_{sa}, x_0, V, N_c\}
\end{aligned}
$$

where $g_j(\mathbf{x})$ are the constraints given in the previous section and $\mathbf{x}$ is the vector of design variables. The autonomous optimization methodology to solve this nonlinear constrained optimization problem is given in the following sections.

A. EMO Approach Using NSGA-II

EAs are non-deterministic (stochastic) methods that mimic evolutionary principles, e.g. natural selection and the survival of the fittest, to constitute their optimization strategy. They work with a set of solutions (population) instead of a single point as in traditional (classical) methods and this gives an opportunity to attack a complex problem (discontinuous, noisy, multi-modal, etc.) in different directions allowing the algorithm to explore as well as exploit the search space. This capability gives an advantage for having a more robust search strategy as compared to traditional algorithms. Since they don't need any gradient information, they are very suitable for black-box (e.g. commercial software) optimization applications. Besides their relatively easy computational implementation, they are also proper for distributed computing applications since all individuals (designs) can be computed independently. Due to their population based search strategy, they have been more popular for the MOPs, often having conflicting objectives resulting not only in a single optimum solution, but in a set of trade-off solutions (Pareto-optimal set), for the last two decades. Many EMO algorithms have been developed in order to solve multiple conflicting goals in an ideal way that is without using weightings between objectives or any scalarization techniques. Besides convergence, a well-spread distribution of these solutions across the Pareto-front has been considered to be an important challenge for MOEAs [16].

As mentioned previously, NSGA-II, which is an EMO algorithm [15] enabling finding well-spread multiple Pareto-optimal solutions for an MOP by incorporating three substantial features, i.e. elitism, non-dominated sorting, and diversity preserving mechanism (crowding distance), is used for the proposed constrained problem.

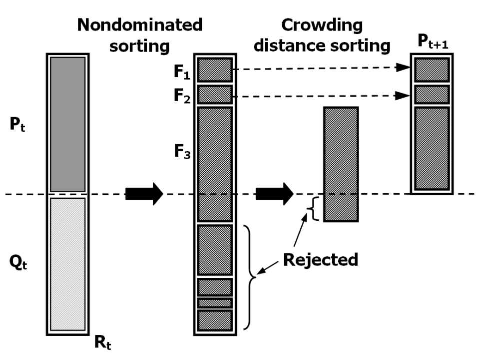

Figure 2. Schematic view of the NSGA-II procedure [15].

Population size is 200 and the number of generations is set to 300 due to relatively high complexity of the objective landscape as shown in Fig. 3. Real variable-coding is used for the design variables. Therefore the simulated binary crossover (SBX) and the polynomial mutation [16], with distribution indices of 5 and 10, are used as a crossover and mutation operators, respectively. Inset plot in Fig. 3 shows all NSGA-II (non-dominated) solutions composing a convex Pareto-optimal front, having V on the horizontal axis and A_{total} on the vertical axis (A_{total} is multiplied by 10^9 to have similar magnitudes in both objectives). As expected, the smaller the required power results in wider comb drive indicating higher number of comb drive fingers. More detailed analysis of these trade-off designs is performed after the local search procedure which aims for further improvement in the convergence of the obtained trade-off frontier.

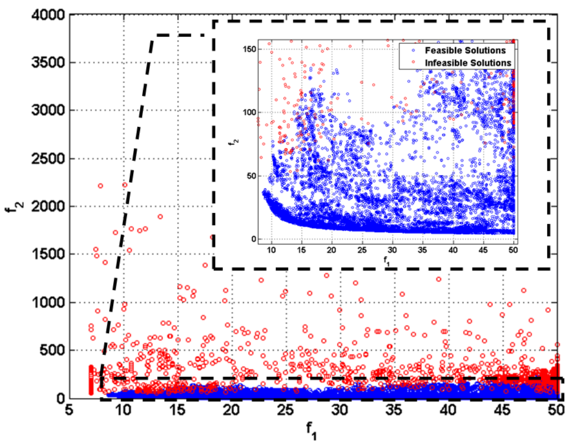

Figure 3. Feasible (blue) and infeasible (red) solutions found so far after 300 generations using NSGA-II (pop. size is 200). The horizontal axis is (f_1): minimization of volume and the vertical axis is (f_2): minimization of total area.

The Pareto-optimal front (non-dominated front) is composed of 199 solutions and is shown in Fig. 4.

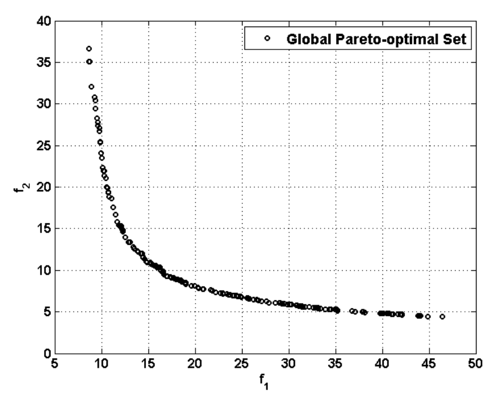

Figure 4. Pareto-optimal solutions (The number of members is 199).

B. Clustering and Local Search

Prior to the local search step, the non-dominated solutions found so far are clustered simply based on their Euclidean distances (i.e. minimum d_i) with respect to their mean, which is computed in each cell and in each axes, in a prefix grid structure to reduce the number of the solutions (for the sake of computational cost), as represented in Fig. 5 on a hypothetically distributed points in the objective space. Fig. 6 shows 18 clustered solutions, indicated by cross markers, out of 199 non-dominated solutions for the FSW problem in a 10-by-10 grid.

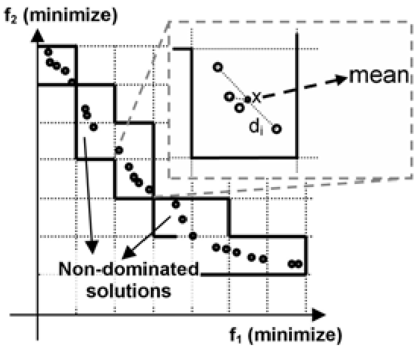

Figure 5. Clustering scheme [22].

Although evolutionary computation has become an important problem solving methodology with its population-based collective learning process, self-adaptation, and robustness, the performance of the algorithms still depend on proper selection of various parameters (e.g. probabilities, selection and mutation schemes, etc.), namely the proper relationship between the exploration and the exploitation capabilities avoiding premature convergence, as mentioned above. Moreover, the computational speed is relatively slow as compared to the classical (deterministic) algorithms. Therefore, as expected, the need for hybrid algorithms, which combine an evolutionary algorithm with e.g. a local search method, emerges aiming at both robust and accurate solutions with (if possible at all) less computational cost. Local search methods may be incorporated within the population members (parents) or among the offspring. The architectures of hybrid evolutionary algorithms have been summarized by [17] as follows: hybridization between two different EAs (a GP assisted GA), an EA with a neural network, a particle swarm optimization (PSO) or an ant colony optimization (ACO) as well as hybridization between EA and other heuristics (such as local search, tabu search, simulated annealing, hill climbing, etc.). However, as the *No Free Lunch* Theorem proposes, on average, all black-box algorithms have identical behavior, thus there is not a definite answer for which local search procedure to use for any sort of problems.

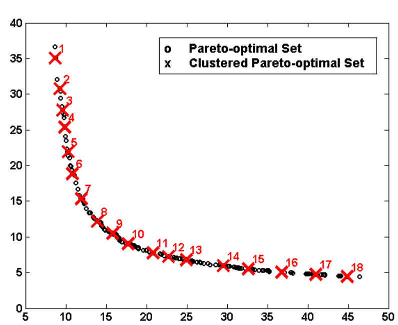

Figure 6. Clustered non-dominated solutions indicated by crosses and the corresponding ID-numbers on top of them.

In the next step, before investigating some common design principles for the layout of MEMS device among the members of the Pareto-optimal set, NSGA-II solutions are sought to be further improved or at least validated to be true Pareto-optimum solutions. ε-constraint method [19] is a very suitable approach to alleviate the difficulties faced in scalarization of MOPs. In this approach, the MOP is reformulated by just keeping one of the multiple objectives as an equality constraint and restricting the rest of the objectives within user-specified values (i.e. V is transformed into a constraint by considering ε=10 ^{6} in the present case). More details regarding are given in [16, 19]. In order to solve the scalarized optimization problem by means of searching the minimum of the aggregated objective function, a gradient-based local search technique, i.e. sequential quadratic programming (SQP; **`fmincon`** function in MATLAB Optimization Toolbox), is used. The modified front is shown with the blue curve in Fig. 7. The change in the convergence is not exaggerated, but on the other hand, it enhances the confidence in the trade-off front obtained by NSGA-II.

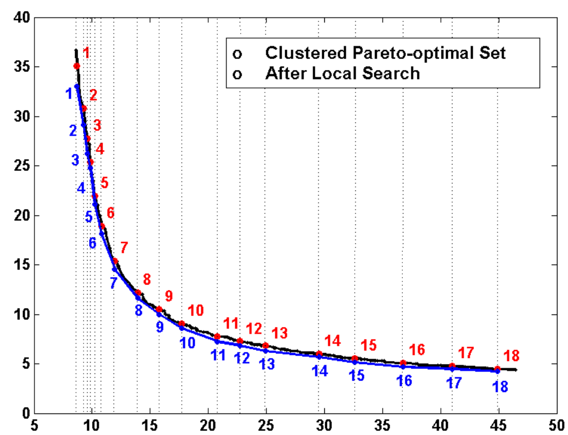

Figure 7. Pareto-optimal front (black curve) modified after the local search (blue curve) on each clustered non-dominated solutions. The ID-numbers of the clustered members and their modified positions are given in red and blue numbers, respectively. Dashed lines show the fixed V values (first objective in MOP becomes an equality constraint) for each independent single objective optimization as proposed the proposed methodology of the ε-constraint method.

C. *Post-optimality Analysis: Discovering Design Principles*

After completing the multi-objective optimization task, a set of optimal solutions specifying the design variables and their trade-offs is obtained. If these optimal solutions are sorted according to the worse order of the first objective (min. V), they would also get lined up in the second objective (min. A_{total}) in an ascending order. Having such a wide variety of solutions make the decision making process much easier compared to having only one optimal solution. This enables engineers or designers to judge or plan the performance of a product or a process in a larger perspective in terms of sacrifices and gains with respect to multiple criteria [16]. Moreover, a basic post-optimality study can unveil interesting design knowledge that is common to all of these trade-off solutions or a partial set of them [20]. This design methodology that is originally formulated as “innovization” (innovation through optimization)

[21, 25], has also been applied manually to the current MEMS design problem.

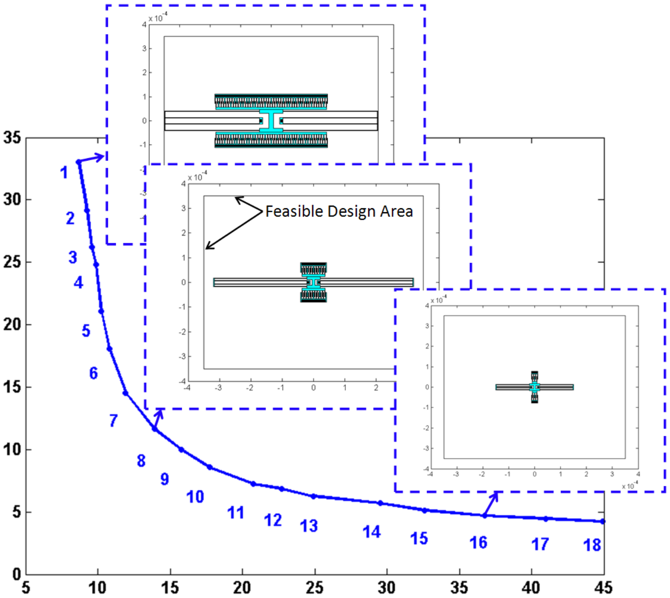

Figure 8. Different layouts on the Pareto-optimal front.

Fig. 8 shows some of the designs out of 18 layout possibilities found so far. The designs are lined up, from left to right, in decreasing size of the device as mentioned above. Even though each structure has a different size, there are some features not changing from one design to the other one. In order to unveil this interesting design knowledge or trends that is common to all of these trade-off solutions or a partial set of them in a more quantitative way, each design parameter is drawn separately with respect to the designs which are sorted with respect to first objective (horizontal axis) from the minimum to the maximum. Since the modified non-dominated front is very close to the one that is obtained by NSGA-II, it is preferred to plot the designs which are obtained in the EMO calculation, not in the local search. This would help to investigate more global trends (since the size of the whole Pareto-set is larger than the clustered set) for the layout design problem. It is very interesting to see in Figs. 9 through 15 that there are indeed some common design principles in some of the design parameters while in some of them there is too much scatter which makes it difficult to approximate aquantitative behavior.

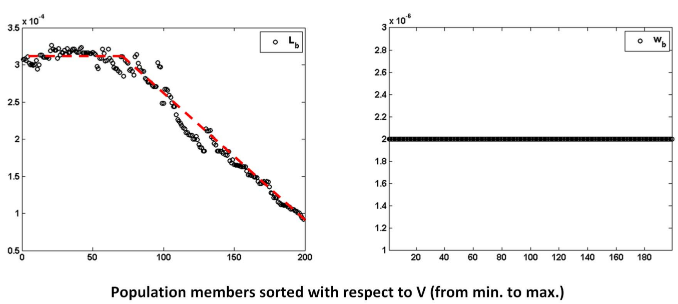

Figure 9. L_b (μm) and w_b (μm) design parameters vs. Pareto-optimal members which are sorted with respect to V (from min. to max.).

Fig. 9 shows the distribution of L_b and w_b among Pareto-optimal members. Two major regions can be seen from the distribution of Lb which is distinguished with red dashed lines. The same procedure for showing similar trends is followed in the next figures. On the other hand, as seen from the graph on the right side in Fig. 9, the points for the design parameter w_b shows a clear distribution along 2 μm. This hidden know-how is very important for designers or manufacturers that, for instance, it will allow them to be able to manufacture different "optimal" designs (i.e. since all these trade-off designs are optimal) in mass production while keeping some the parameters fixed without sacrificing in the optimality criteria.

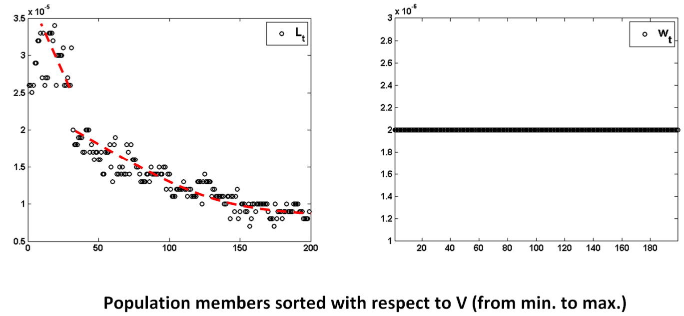

Figure 10. L_t (μm) and w_t (μm) design parameters vs. Pareto-optimal members which are sorted with respect to V (from min. to max.).

Fig. 10 shows the distribution of L_t and w_t among Pareto-optimal members. There are again similar trends in these two graphs as compared to Fig. 9, in which w_t shows a clear distribution along 2 μm and there are two main regions for L_t distinguished with two red dashed lines.

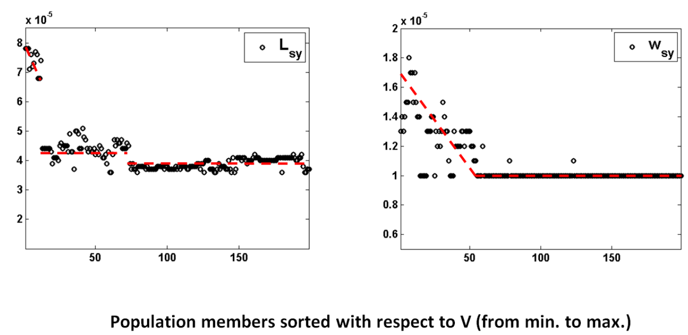

Figure 11. L_{sy} (μm) and w_{sy} (μm) design parameters vs. Pareto-optimal members which are sorted with respect to V (from min. to max.).

Fig. 11 and 12 show the distribution of L_{sy} and w_{sy} as well as w_{sa} and w_{cy} among Pareto-optimal members, respectively.

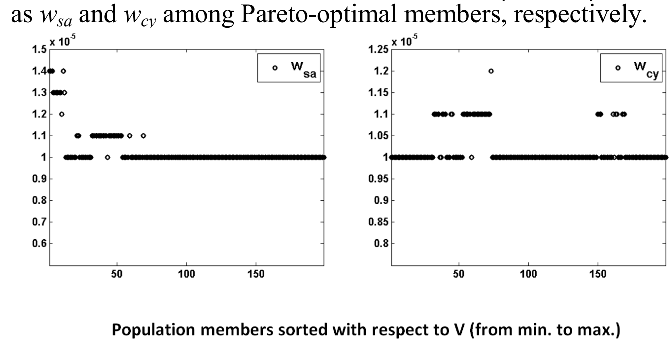

Figure 12. w_{sa} (μm) and w_{cy} (μm) design parameters vs. Pareto-optimal members which are sorted with respect to V (from min. to max.).

Fig. 13 shows the distribution of L_c and w_c among Pareto-optimal members. It is not clear enough to approximate a quantitative description for L_c, but on the other hand w_c is fixed at 2 μm.

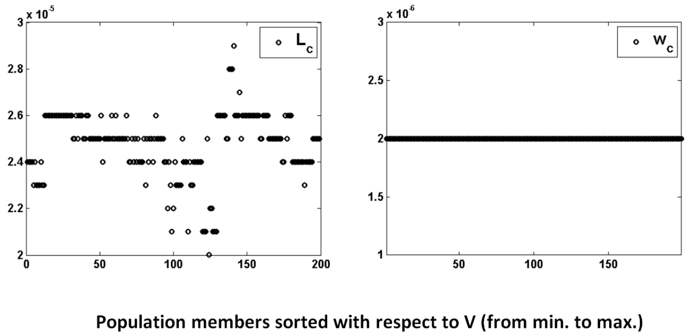

Figure 13. L_c (μm) and w_c (μm) design parameters vs. Pareto-optimal members which are sorted with respect to V (from min. to max.).

The distribution of L_{sa} and x_0 is shown in Fig. 14.

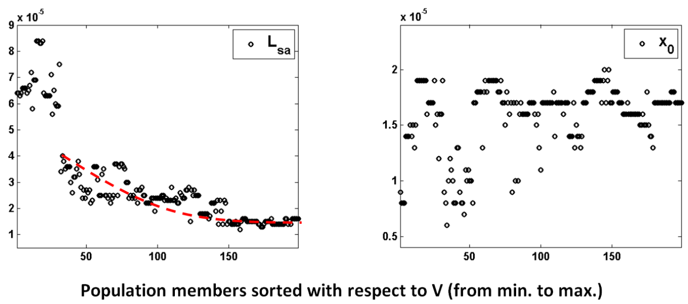

Figure 14. L_{sa} (μm) and x_0 (μm) design parameters vs. Pareto-optimal members which are sorted with respect to V (from min. to max.).

Finally, the V and N_c parameters are shown in Fig. 15. Four regions (3 linear curves and 1 nonlinear curve) distinguishing the design trends in N_c parameter is clearly seen.

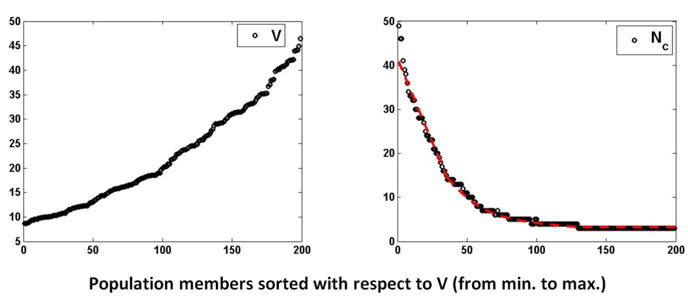

Figure 15. V (Volt) and N_c design parameters vs. Pareto-optimal members which are sorted with respect to V (from min. to max.).

It is worthwhile to emphasize that these "hidden" trends, which are explored via mapping from the objective landscape into the design space, play a vital role for the designers and manufacturers not only in the field of MEMS, but also in other fields of engineering applications, e.g. [22].

In specific to this design problem, the designs 1 and 16 in Fig. 8 seems to be reasonable choices with respect to available manufacturing technology or cost limitations. The size of the design-16 is very small as compared to the design-1, but on the other hand the power it consumes is also considerably larger.

IV. CONCLUSIONS AND DISCUSSIONS

This paper introduces a memetic computational methodology for the autonomous investigation in a multi-objective optimization problem for the design of MEMS components. More specifically, a case study of layout synthesis of a comb-driven micro-resonator shows that the approach proposed in this paper can lead to design results accommodating two design objectives, i.e. simultaneous minimization of size and power input (voltage) of a MEMS device.

The major contribution of this paper is the application of memetic computing in the field of MEMS design. An EMO technique, in particular NSGA-II, has been applied to find multiple trade-off solutions followed by a gradient-based local search, i.e. SQP, to improve the convergence of the obtained Pareto-optimal front. In order to reduce the number of function evaluations in the local search procedure, the obtained non-dominated solutions are clustered in the objective space. It is found out that the non-dominated front obtained by NSGA-II is very close to the one obtained after the local search. Even though the improvement in the Pareto-optimal set is negligible, this still gives a confidence in the optimal designs to continue the investigation further on with a post-optimality study which enables users to unveil some common design principles among this set of optimal solutions. Finally, two reasonable design choices have been offered based on manufacturability issues.

This post-optimality study, which has originally been entitled as "innovization", has been manually applied in this particular work. Automation of this procedure as similar to the one followed in the solution of the MOP is one of the future challenges to be tackled.

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