EVOLUTIONARY MULT- AND MANY-OBJECTIVE OPTIMIZATION ALGORITHM USING REGION DECOMPOSITION

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Problem Description

Multi-Objective Optimization Problem (MOP):

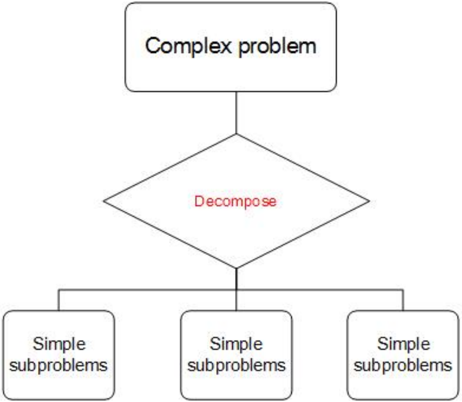
$$\min F(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_M(\mathbf{x}))^T$$
$$s.t.\ \mathbf{x} \in \Omega.$$

where

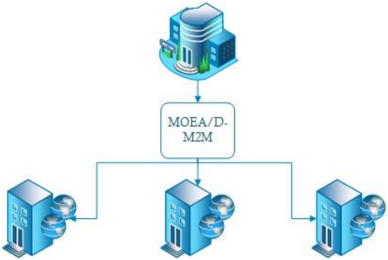
Ω: the decision space,
x: decision variable/solution,
f_i: $\mathbf{D} \rightarrow \mathbf{R}^m$, objective function,
R^m: objective space.

Note: When $m > 3$, it is called a Many objective Optimization Problem (MaOP).

Why region decomposition?

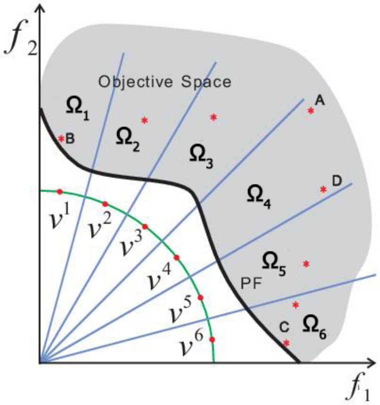


The results of region decomposition



- All of subproblems have independence and can be solved in a collaborative way. It makes convergence and distribution are equally important. Not convergence first.
- Each subproblem has its own subpopulation, and genetic operators can be conducted in those subpopulation independently.

MOEA/D-M2M Region Decomposition



- MOEA/D-M2M divides $\mathbf{R}_+^m$ into K subregions $\Omega_1, \ldots, \Omega_K$ by K direction vectors $\mathbf{v}^1, \ldots, \mathbf{v}^K$ in $\mathbf{R}_+^m$. Where

$$\Omega_k = \{\mathbf{u} \in \mathbf{R}_+^m | \langle \mathbf{u}, \mathbf{v}^k \rangle \leq \langle \mathbf{u}, \mathbf{v}^j \rangle \text{ for any } j = 1, \ldots, K\},$$

$\langle \mathbf{u}, \mathbf{v}^j \rangle$ is the acute angle between $\mathbf{u}$ and $\mathbf{v}^j$.

Main Idea of MOEA/D-M2M

- MOEA/D-M2M is a new version of MOEA/D, which decomposes an MOP into a set of simple multiobjective optimization subproblems.
- MOEA/D-M2M solves these subproblems in a collaborative way, and it can reduce the computational effort at each generation.
- MOEA/D-M2M has a strong ability in maintaining population diversity, which is critical for solving some MOPs.

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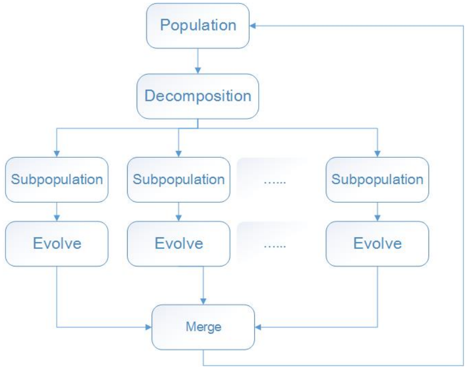
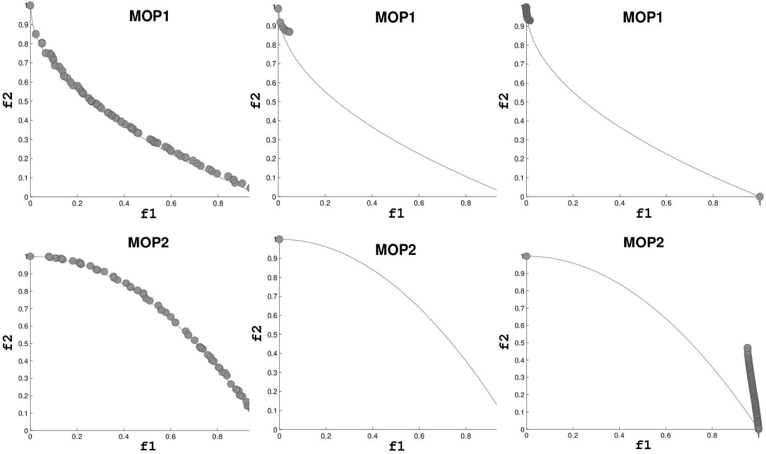
The flow diagram of MOEA/D-M2M



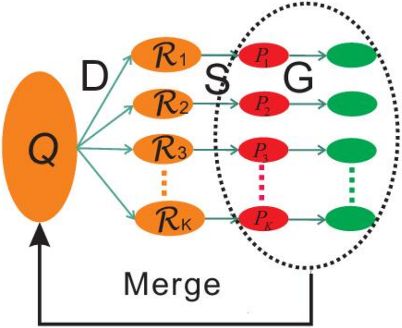
- MOEA/D-M2M decomposes an MOP into a set of simple multiobjective optimization subproblems.
- Those multiobjective optimization subproblems are then solved in a collaborative way.

[1] H.L. Liu, Fangqing Gu and Q. Zhang, Decomposition of a multiobjective optimization problem into a number of simple multiobjective subproblems, IEEE Transactions on Evolutionary Computation, 18(3):450-455, 2014.

Plot of the nondominated front found by MOEA/D-M2M, MOEA/D-DE and NSGA-II.

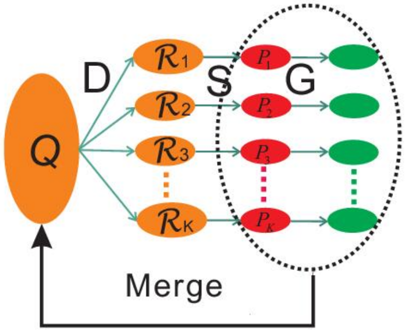


Why Hybridizing Different EMOAs?



- Each EMOA has its own strengths and weaknesses according to "no free lunch" theorems.
- Hybridizing different EMOAs in MOEA/D-M2M framework is a very natural way to deal with hard MOPs.

Why Hybridizing Different EMOAs?



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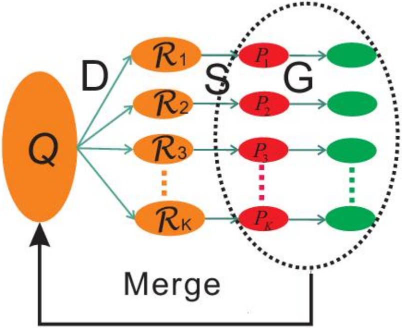
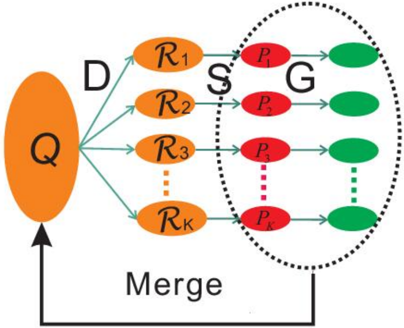
Hybrid Evolutionary Multiobjective Optimization Algorithm (HEMOA)



- Population Q is decomposed into K sets $\mathcal{R}_1$, $\mathcal{R}_2$,...,$\mathcal{R}_K$ by MOEA/D-M2M.
- $\mathcal{P}_k$ is the subpopulation evolved from $\mathcal{R}_k$ by different EMOAs $(k = 1, ..., K)$.

[2] F. Gu, H-L. Liu, and K. C. Tan, A hybrid evolutionary multiobjective optimization algorithm with adaptive multi-fitness assignment, Soft Computing, vol. 19, no. 11, pp. 3249 - 3259, 2015.

Hybrid Evolutionary Multiobjective Optimization Algorithm (HEMOA)



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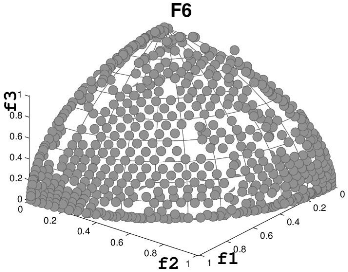
Simulation Results



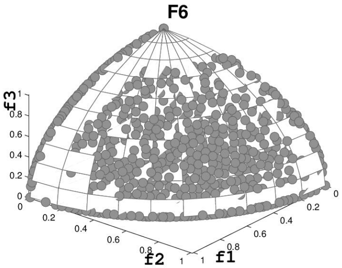


Figure 1.1: Plots of the nondominated front with the best S-metric found by HEMOA and SMS-EMOA.

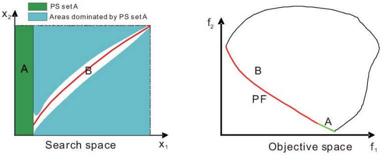
Definition of Imbalanced Problems

Definition 1 (Imbalanced problem)

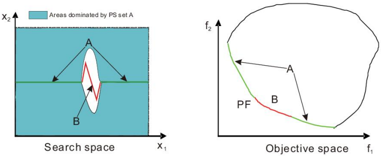
A multi-objective optimization problem is defined to be an imbalanced MOP, if a specific subset (referred here as the 'favored' subset) of its Pareto front (PF) satisfies the following conditions:

1. *The complexity of optimization subproblem corresponding to the 'favored' subset is significantly smaller than the subproblem corresponding to the other part (the 'unfavored' subset) of the PF, and*
2. *The Pareto-Set (PS) of the favored subset dominates a significantly larger part of the feasible variable space than the PS of the unfavored subset.*

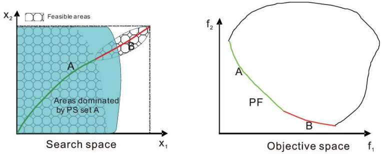
Illustration of the Imbalanced Problems



(a) Imbalanced mapping



(b) Variable Linkages



(c) Constraint Isolation

(d) More to be found

Solve imbalanced Problems by MOEA/D-M2M

Problem 2-1:

$$\text{Min: } \begin{cases} f_1(\mathbf{x}) = (\mathbf{1}+\mathbf{g}(\mathbf{x}))\mathbf{x_1}, \\ f_2(\mathbf{x}) = (\mathbf{1}+\mathbf{g}(\mathbf{x}))(\mathbf{1}-\sqrt{\mathbf{x_1}}), \end{cases}$$

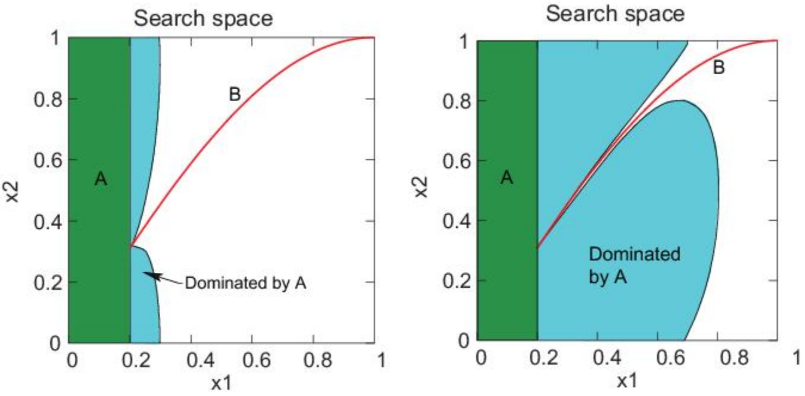
$$\text{where } \begin{cases} g(\mathbf{x}) = \begin{cases} 0 & \text{if } 0 \le x_1 \le 0.2, \\ 0.5(-0.9t^2+|t|^{0.6}) & \text{otherwise,} \end{cases} \\ t = x_2 - \sin(0.5\pi x_1), x_i \in [0,1], i = 1,2. \end{cases}$$

Problem 2-2:

$$\text{Min: } \begin{cases} f_1(\mathbf{x}) = (\mathbf{1}+\mathbf{g}(\mathbf{x}))\mathbf{x_1}, \\ f_2(\mathbf{x}) = (\mathbf{1}+\mathbf{g}(\mathbf{x}))(\mathbf{1}-\sqrt{\mathbf{x_1}}), \end{cases}$$

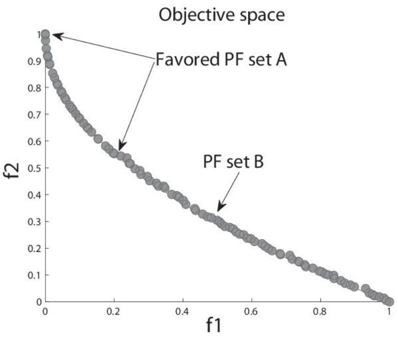
$$\text{where } \begin{cases} g(\mathbf{x}) = \begin{cases} 0 & \text{if } 0 \le x_1 \le 0.2, \\ 10(-0.9t^2+|t|^{0.2}) & \text{otherwise,} \end{cases} \\ t = x_2 - \sin(0.5\pi x_1), x_i \in [0,1], i = 1,2. \end{cases}$$

Search space of Problem 2-1 and Problem 2-2

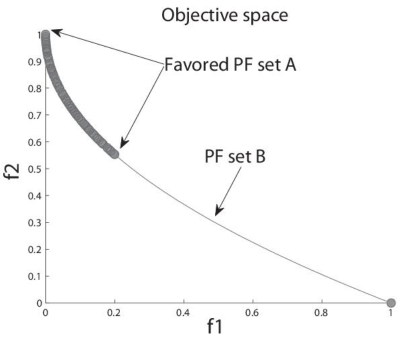


(e) Problem 2-1 (f) Problem 2-2

Obtained non-dominated front by NSGA-II for Problem 2-1 and Problem 2-2

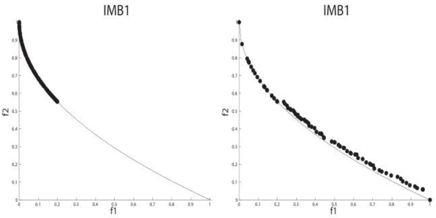


(g) Problem 2-1



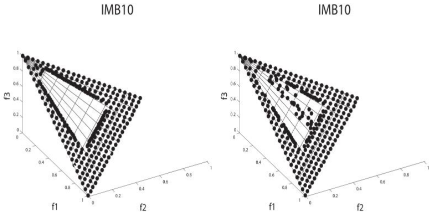
(h) Problem 2-2

Simulation Results of Imbalanced Problems #1



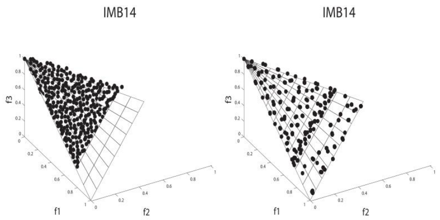
- Obtained non-dominated solutions for the median IGD run using NSGA-II (the left panel) and NSGA-II-M2M (the right panel) for Imbalanced Problem IMB1, and the Imbalance is caused by Imbalanced mapping.

Simulation Results of Imbalanced Problems #2



- Obtained non-dominated solutions for the median IGD run using MOEA/D (the left panel) and MOEA/D-M2M (the right panel) for Imbalanced Problem IMB10, and the Imbalance is caused by Variable Linkages.

Simulation Results of Imbalanced Problems #3



- Obtained non-dominated solutions for the median IGD run using EMOA-SMS (the left panel) and SMS-M2M (the right panel) for Imbalanced Problem IMB8, and the Imbalance is caused by Constraint Isolation.

[3] H-L. Liu, L. Chen, K. Deb, and E. D. Goodman, Investigating the effect of imbalance between convergence and diversity in evolutionary multiobjective algorithms, IEEE Trans. Evol. Comput.,Vol. 21(3): pp.408-425, 2017.

Multiple-objective 0/1 knapsack problem (MOKP) #4

Multiple-objective 0/1 Knapsack Problem (MOKP) is a kind of classic combination optimization problems, which can be used to model a lot of application problems.

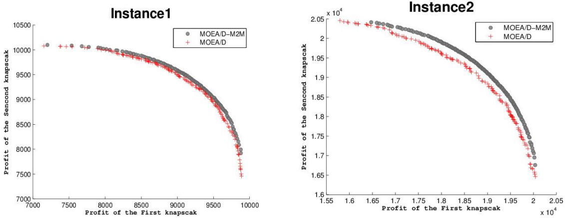


Figure 1.2: Plot of the nondominated front found by MOEA/D-M2M and MOEA/D-DE.

Multiple-objective Tracking Area Planning (MTAP) Problem #5

Planning the Tracking Area (TA) for an Long Term Evolution (LTE) network is to minimize of both location update cost and paging cost.

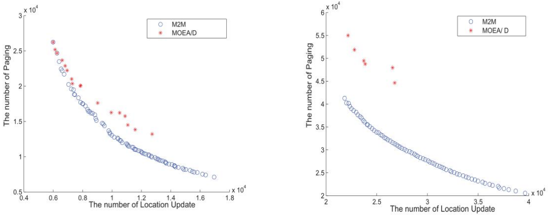
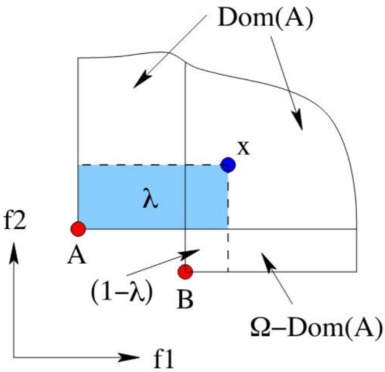


Figure 1.3: Plot of the nondominated front found by MOEA/D-M2M and MOEA/D-DE for the 5×6 network and 9×9 network.

[4] L. Chen, H-L. Liu, Z. Fan, S. Xie and E. D. Goodman, Modeling the tracking area planning problem using an evolutionary multi-objective algorithm, IEEE Computational Intelligence Magazine, pp.29-41, Feb., 2017.

Theoretical Study of MOEA/D-M2M



Theorem 1

When optimizing an imbalanced problem, the population of a 'convergence first and diversity second'-selection-strategy-based EMO algorithms is more likely to converge to the favored PF.

[3] H-L. Liu, L. Chen, K. Deb, and E. D. Goodman, Investigating the effect of imbalance between convergence and diversity in evolutionary multiobjective algorithms, IEEE Trans. Evol. Comput.,Vol. 21(3): pp.408-425, 2017.

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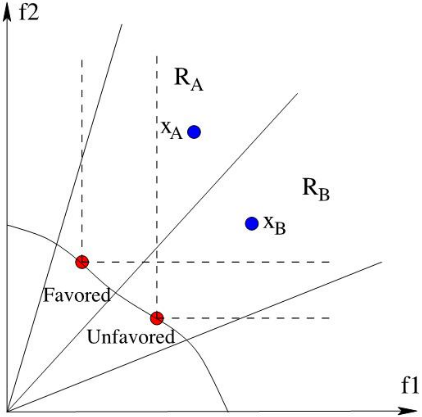

Theorem 2

The M2M population decomposition strategy can effectively aid EMO algorithms to converge into both favored and unfavored PFs when optimizing imbalanced problems.

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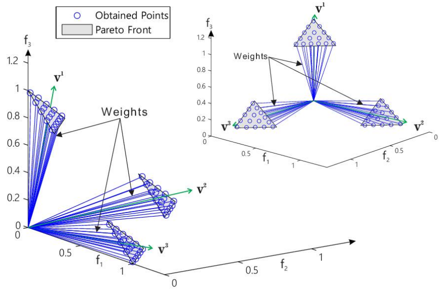
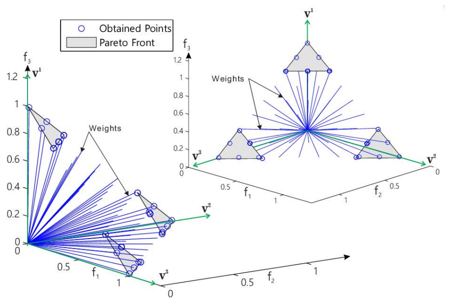
Why adaptive decomposition?



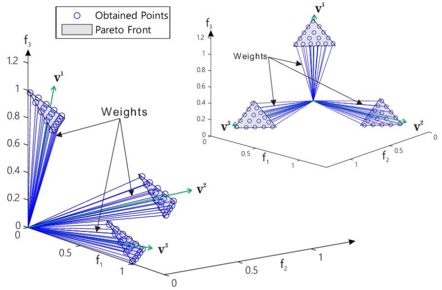
- In MOEA/D-M2M, a set of direction vectors need to be defined in the initialization, if regions are divided in a proper way and weight vectors are selected in a right way, then MOEA/D-M2M can solve the problem we want to solve.

Why adaptive decomposition?



- However, the truth is that a MOP's PF may be unknown before optimizing it. Most of the weight vectors aided EMO algorithms have no alternatives but to adopt the evenly distributed weight vectors design. And it may not work well.

Why adaptive decomposition?



- If the weight and direction vectors are designed according to PF shape, we can obtained a set of well distributed Pareto optimal points on the PF.

Main Idea of MOEA/D-AM2M

1. As an improvement of MOEA/D-M2M, MOEA/D-AM2M periodically resets the subregion setting and weight vectors setting.
2. The adaptive design of subregion and weights is based on the distribution of the current solutions.
3. MOEA/D-AM2M can effectively avoid the waste of search effort and focus on more promising regions.

[5] H-L. Liu, L. Chen, Q. Zhang, and K. Deb, Adaptively allocating search effort in challenging many-Objective optimization problems, IEEE Trans. Evol. Comput.,Vol. 22(3): pp.433-448, 2018.

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Adaptive Direction Vectors Design

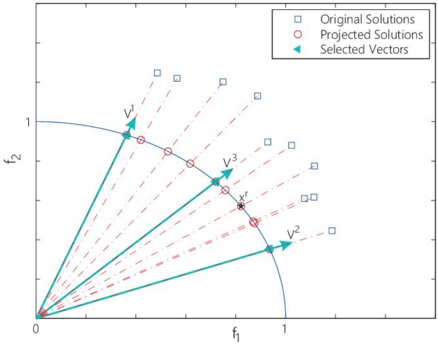

Figure 2.1: Illustration of the direction vectors design.

- A vector x^r is firstly selected, vector v^1 having the largest acute angle with x^r is selected as the first direction vector and then the vector having the largest acute angle with v^1 is selected as direction vector v^2. Vector v^3 is selected to make it such a direction which has the maximum of minimum the angle between v^3 and v^1 and v^3 and x^2.

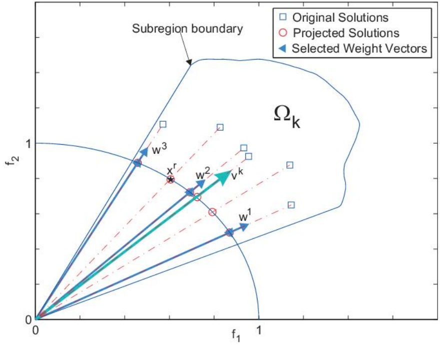

Figure 2.2: Illustration of the weight vectors design.

- Vector x^r is also the randomly selected, the firstly selected weight vector w^1 have the smallest cosine similarity with the randomly selected vector x^r. The weight vectors w^2 and w^3 are then selected similarly.

Definition of Degenerated MaOPs

Definition 1 (Degenerated MaOP)

If the dimension of a MaOP with m objectives is less than $m-1$, we call this optimization problem a degenerated MaOP.

- Under mild conditions, the PF are $(m-1)$-D piecewise continuous manifolds.
- Degenerated MaOP can be regarded as MaOP with low dimension PFs.
- Identification the importance of different objectives and determination of how the search effort to be allocated to the search space is very important.

Generation of Degenerated MaOPs

$$\text{Minimize} \begin{cases} f_1(\mathbf{x}) = (\mathbf{1} + \mathbf{g}(\mathbf{x_{II}}))\mathbf{t_1}(\mathbf{x_I}) \\ f_2(\mathbf{x}) = (\mathbf{1} + \mathbf{g}(\mathbf{x_{II}}))\mathbf{t_2}(\mathbf{x_I}) \\ \dots \\ f_k(\mathbf{x}) = (\mathbf{1} + \mathbf{g}(\mathbf{x_{II}}))\mathbf{t_k}(\mathbf{x_I}) \\ f_i(\mathbf{x}) = (\mathbf{r_{i1}f_1}(\mathbf{x}) + \mathbf{r_{i2}f_2}(\mathbf{x}) + \dots + \mathbf{r_{ik}f_k}(\mathbf{x}))\mathbf{g_i} \end{cases}$$

where
$k + 1 \leq i \leq m$, $\mathbf{x} = (\mathbf{x_1}, \mathbf{x_2}, \dots, \mathbf{x_n})$, $\mathbf{x_I} = (\mathbf{x_1}, \mathbf{x_2} \dots, \mathbf{x_d})$, $\mathbf{x_{II}} = (\mathbf{x_{d+1}}, \mathbf{x_{d+2}}, \dots, \mathbf{x_n})$. Its PF and PS are $d - 1$ and $n - d - 1$ manifolds respectively. $g(\mathbf{x_{II}}) \geq 0$ is used to control the distribution of PS, and $g_i \geq 0$ is related to $g(\mathbf{x_{II}})$. If $g(\mathbf{x_{II}}) = 0$, then we can get $g_i = 0$ ($\forall i = k + 1, \dots, m$), but not vice versa.

Simulation Results of Five Imbalanced Problems

Table 1: The best and mean of IGD-metric values of MOEA/D-AM2M and MOEA/D-DE in 20 independent runs for each test instance

IGD-metric	MOEA/D-AM2M		MOEA/D-DE	
Instance	best	mean	best	mean
MaOP1	**0.002286**	**0.002649**	0.045839	0.054036
MaOP2	**0.002813**	**0.004250**	0.232158	0.232545
MaOP3	**0.016442**	**0.017029**	0.094145	0.098302
MaOP4	**0.023468**	**0.025032**	0.115029	0.119855
MaOP5	**0.020565**	**0.021155**	0.104285	0.107219

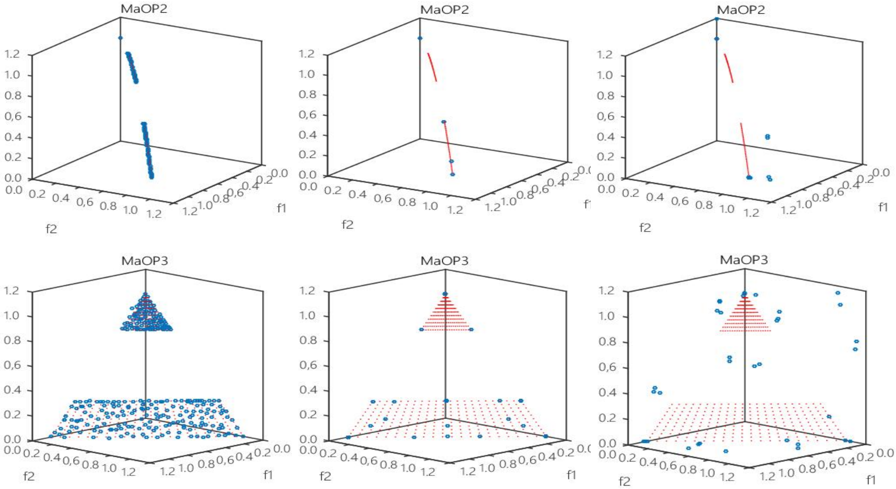

Figure 2.3: Plot of the solutions with the median HV-metric value found by MOEA/D-AM2M, MOEA/D-DE and NSGA-III-AM2M for test problem 2-3.

CONTENTS

Conclusions

- Conclusion
  - Proposed the MOEA/D-M2M population decomposition framework.
  - Improved MOEA/D-M2M to MOEA/D-AM2M with adaptively subregion decomposition and weight vectors design.
  - Studied the performance of MOEA/D-M2M and MOEA/D-AM2M.

Future Work

- Future Work
  - Conduct more experiments to show the stability of the proposed algorithm.
  - Investigate the performance of MOEA/D-AM2M on solving generally MaOPs test problems.
  - Investigate the performance of MOEA/D-AM2M on solving some practical application MaOPs.

- H-L. Liu, F. Gu, and Q. Zhang, "Decomposition of a multiobjective optimization problem into a number of simple multiobjective subproblems," *IEEE Trans. Evol. Comput.*, vol. 18, no. 3, Jun. 2013.
- F. Gu, H-L. Liu, and K. C. Tan, "A hybrid evolutionary multiobjective optimization algorithm with adaptive multi-fitness assignment," *Soft Computing*, vol. 19, no. 11, pp. 3249–3259, 2015.
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Thank you for your attention!