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PII: S2210-6502(18)30023-3
DOI: 10.1016/j.swevo.2018.08.017
Reference: SWEVO 440

To appear in: Swarm and Evolutionary Computation BASE DATA

Received Date: 12 January 2018
Revised Date: 27 June 2018
Accepted Date: 28 August 2018


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Push and Pull Search for Solving Constrained Multi-objective Optimization Problems

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Abstract

This paper proposes a push and pull search (PPS) framework for solving constrained multi-objective optimization problems (CMOPs). To be more specific, the proposed PPS divides the search process into two different stages: push and pull search stages. In the push stage, a multi-objective evolutionary algorithm (MOEA) is used to explore the search space without considering any constraints, which can help to get across infeasible regions very quickly and to approach the unconstrained Pareto front. Furthermore, the landscape of CMOPs with constraints can be probed and estimated in the push stage, which can be utilized to conduct the parameter setting for the constraint-handling approaches to be applied in the pull stage. Then, a modified form of a constrained multi-objective evolutionary algorithm (CMOEA), with improved epsilon constraint-handling, is applied to pull the infeasible individuals achieved in the push stage to the feasible and non-dominated regions. To evaluate the performance regarding convergence and diversity, a set of benchmark CMOPs and a real-world optimization problem are used to test the proposed PPS (PPS-MOEA/D) and state-of-the-art CMOEAs, including MOEA/D-IEpsilon, MOEA/D-CDP, MOEA/D-SR, C-MOEA/D and NSGA-II-CDP. The comprehensive experimental results show that the proposed PPS-MOEA/D achieves significantly better performance than the other six CMOEAs on most of the tested problems, which indicates the superiority of the proposed PPS method for solving CMOPs.

Keywords: Push and Pull Search, Constraint-handling Mechanisms, Constrained Multi-objective Evolutionary Algorithms, Multi-objective Evolutionary Algorithm Based on Decomposition (MOEA/D)

1. Introduction

Many real-world optimization problems can be summarized as optimizing a number of conflicting objectives simultaneously with a set of equality and/or inequality constraints. Such problems are called constrained multi-objective optimization problems (CMOPs). Without loss of generality, a CMOP considered in this paper can be defined as follows: \begin{equation}
\begin{aligned}
\text{minimize} & \quad F(x) = (f_1(x), \ldots, f_m(x))^T \\
\text{subject to} & \quad g_i(x) \geq 0, \quad i = 1, \ldots, q \\
& \quad h_j(x) = 0, \quad j = 1, \ldots, p \\
& \quad x \in \mathbb{R}^n
\end{aligned}
\end{equation}

where $F(x) = (f_1(x), f_2(x), \ldots, f_m(x))^T$ is an $m$-dimensional objective vector, and $F(x) \in \mathbb{R}^m$. $g_i(x) \geq 0$ is an inequality constraint, and $q$ is the number of inequality constraints. $h_j(x) = 0$ is an equality constraint, and $p$ represents the number of equality constraints. $x \in \mathbb{R}^n$ is an $n$-dimensional decision vector. When solving CMOPs with inequality and/or equality constraints, we usually convert the equality constraints into inequality constraints by introducing an extremely small positive number $\delta$. The detailed transformation is given as follows:

\begin{equation}
\begin{aligned}
\phi(x) = \sum_{i=1}^q |\min(g_i(x), 0)| + \sum_{j=1}^p |\min(h_j(x), 0)|
\end{aligned}
\end{equation}

Given a solution $x^k \in \mathbb{R}^n$, if $\phi(x^k) = 0$, $x^k$ is feasible. All the feasible solutions constitute a feasible solution set $S$, which is defined as $S = \{x|\phi(x) = 0, x \in \mathbb{R}^n\}$. For any two solutions $x^a, x^b \in S$, $x^a$ is said to dominate $x^b$ if $f_i(x^a) \leq f_i(x^b)$ for each $i \in \{1, \ldots, m\}$ and $f_i(x^a) < f_i(x^b)$ for at least one $j \in \{1, \ldots, m\}$, denoted as $x^a \preceq x^b$. If there is no other solution in $S$ dominating solution $x^a$, then $x^a$ is called a Pareto optimal solution. All of the Pareto optimal solutions constitute a Pareto optimal set ($PS$). The mapping of the $PS$ in the objective space is called a Pareto optimal front ($PF$), which is defined as $PF = \{F(x)|x \in PS\}$.

A key issue in CMOEAs is to maintain a balance between minimizing the objectives and satisfying the constraints. In fact, most constraint-handling mechanisms in evolutionary computation are designed to try to achieve this balance. For example,
the penalty function approach adopts a penalty factor \( \lambda \) to maintain the balance between minimizing the objectives and satisfying the constraints. It converts a CMOOP into an unconstrained MOP by adding the overall constraint violation multiplied by a predefined penalty factor \( \lambda \) to each objective \([2]\). In the case of \( \lambda = \infty \), it is called a death penalty approach \([3]\), which means that infeasible solutions are totally unacceptable. If \( \lambda = 0 \) is a static value during the search process, it is called a static penalty approach \([4]\). If \( \lambda \) is changing during the search process, it is called a dynamic penalty approach \([5]\). In the case in which \( \lambda \) is changing according to the information collected during the search process, it is called an adaptive penalty approach \[9\].

In order to avoid the need to tune the penalty factors, another type of constraint-handling method is also in use, which composes the objectives and constraints separately. Representative examples include the constraint dominance principle \( \text{CDP}[10] \), epsilon constraint-handling method \( \text{EC}[11] \), stochastic ranking approach \( \text{SR}[12] \), and so on. In \( \text{CDP}[10] \), three basic rules are adopted to compare any two solutions. In the first rule, given two solutions \( x, x' \in R^n \), if \( x' \) is feasible and \( x \) is infeasible, \( x' \) is better than \( x \). If \( x' \) and \( x \) are both infeasible, the one with a smaller constraint violation is better. In the second rule, last rule, \( x, x' \) are both feasible, and the one dominating the other is better. \( \text{CDP} \) is a popular constraint-handling method, as it is simple and has no extra parameters. However, it is not suitable for solving CMOOPs with very small and narrow feasible regions \([13]\). For many generations, most or even all solutions in the working population are infeasible when solving CMOOPs with this property. In addition, the diversity of the working population can hardly be well maintained, because the selection of solutions is only based on the constraint violations according to the second rule of \( \text{CDP} \).

In order to solve CMOOPs with small and narrow feasible regions, the epsilon constraint-handling \( \text{EC}[11] \) approach has been suggested. It is similar to \( \text{CDP} \) except for the relaxation of the constraints. In \( \text{EC} \), the relaxation of the constraints is controlled by the epsilon level \( \varepsilon \), which can help to maintain the diversity of the working population in the case when most solutions are infeasible. To be more specific, if the overall constraint violation of a solution is less than \( \varepsilon \), this solution is deemed feasible. The epsilon level \( \varepsilon \) is a critical parameter in \( \text{EC} \). In the case of \( \varepsilon = 0 \), \( \text{EC} \) is the same as \( \text{CDP} \). Although \( \text{EC} \) can be used to solve CMOOPs with small feasible regions, controlling the value of \( \varepsilon \) properly is not at all trivial.

Both \( \text{CDP}[10] \) and \( \text{EC}[11] \) first compare the constraints, then compare the objectives. \( \text{SR}[12] \) is different from \( \text{CDP} \) and \( \text{EC} \) in terms of the order of comparison. It adopts a probability parameter \( p_f \in [0, 1] \) to decide if the comparison is to be based on objectives or constraints. For any two solutions, if a random number is less than \( p_f \), the one with the non-dominated objectives is deemed better—i.e., the comparison is based on objectives. On the other hand, if the random number is greater than \( p_f \), the comparison is based first on the constraints, then on the objectives, as is the case with \( \text{CDP} \). In the case of \( p_f = 0, 1 \), \( \text{SR} \) is equivalent to \( \text{CDP} \).

In recent years, much work has been done in the field of many-objective evolutionary algorithms (MaOEAs) \([14]\), which gives us new ways to solve CMOOPs. In order to balance the constraints and the objectives, some researchers adopt multi-objective evolutionary algorithms (MOEAs) or MaOEAs (when the number of objectives is greater than three) to deal with constraints \([15]\). For an \( M \)-objective CMOOP, its constraints can be converted into one or \( k \) extra objectives. Then the \( M \)-objective CMOOP is transformed into an \((M + 1)\)- or \((M + k)\)-objective unconstrained MOP, which can be solved by MOEAs or MaOEAs. Representative examples include Cai and Wang’s Method (CW) \([16]\) and the feasibility driven evolutionary algorithms (IDEA) \([17]\).

To maintain a good balance between minimizing the objectives and satisfying the constraints, some researchers combine several constraint-handling mechanisms, which can be further divided into two categories, including adopting different constraint-handling mechanisms in either different evolutionary stages or in different subproblems. For example, the adaptive trade-off model \( \text{ATM}[18] \) uses two different constraint-handling mechanisms, including a multi-objective approach and adaptive penalty functions, in different evolutionary stages. The ensemble of constraint-handling methods \( \text{ECHM}[19] \) uses different constraint-handling techniques, including epsilon constraint-handling \( \text{EC}[11] \), self-adaptive penalty functions \( \text{SP}[9] \) and superiority of feasible solutions \( \text{SF}[20] \). Three subpopulations are generated in \( \text{ECHM} \), and each subpopulation uses a different constraint-handling method.

In this paper, we propose a biphasic CMOEA, namely push and pull search \( \text{PPS} \), to balance objective minimization and constraint satisfaction. Unlike the above-mentioned constraint-handling methods, \( \text{PPS} \) divides the search process into two different stages. In the first stage, only the objectives are optimized, which means the working population is pushed toward the unconstrained PF without considering any constraints. Furthermore, the landscape of constraints in CMOOPs can be estimated in the push stage, which can be applied to conduct the parameter setting of the constraint-handling approaches to be applied in the pull stage. In the pull stage, an improved epsilon constraint-handling approach is adopted to pull the working population to the constrained PF. In summary, it provides a new framework and has the following potential advantages.

1. It has the ability to get across large infeasible regions of the constrained PF. Since the constraints are ignored in the push stage, any infeasible regions encountered before the true PF present no barriers for the working population.
2. It facilitates the parameter setting in the constraint-handling methods. Since the landscape of constraints has already been explored by the push process, much information has been discovered and gathered to guide the parameter setting for the pull stage.

The rest of the paper is organized as follows. Section 2 introduces the general idea of \( \text{PPS} \). Section 3 gives an instantiation of \( \text{PPS} \) in the framework of MOEA/D, called PPS-MOEA/D. Section 4 designs a set of experiments to compare the proposed PPS-MOEA/D with six other CMOEAs, including MOEA/D-Epsilon \([21]\), MOEA/D-Epsilon \([22]\), MOEA/D-SR.
Constraints define infeasible regions in the decision space, and sometimes are defined in such a way that they have an effect on the PF in the objective space. The influence of infeasible regions on PFs can be generally classified into three different situations. For each situation, the search behavior of PPS is illustrated by Fig. 13 respectively, which can be summarized as follows.

1. Infeasible regions block the way towards the PF, as illustrated by Fig. 1(a). In this circumstance, the unconstrained PF is the same as the constrained PF, and PPS has significant advantages compared with other CMOEAs. Since the constraints are ignored in the push stage of PPS, the infeasible regions have no effect on the searching of PPS. Figs. 1(a)-(e) show the push process at various stages, showing that the working population crosses the infeasible regions in this case without any extra effort. Because the constrained PF is the same as the unconstrained PF, the true PF has already been approximated by the working population in the push process, so the pull search has no effect on the working population, as shown in Fig. 1(f).

2. The unconstrained PF is covered by infeasible regions and all of it is infeasible. Every constrained Pareto optimal point thus lies on some constraint boundary, as illustrated by Fig. 2(a). In this circumstance, PPS first approaches the unconstrained PF by using the push strategy as illustrated by Fig. 2(a)-(c). After the working population approaches the unconstrained PF, the pull strategy is applied to pull the working population towards the true (constrained) PF, as illustrated by Fig. 2(d)-(f).

3. Infeasible regions make the original unconstrained PF partially feasible, as illustrated by Fig. 3(a). In this situation, some parts of the true PF have already been achieved during the push search, as illustrated by Fig. 3(c). In the pull stage, infeasible solutions are pulled to the feasible and non-dominated regions, as illustrated by Fig. 3(d)-(f). Finally, the entire true PF has been found by PPS. It is worth noting that infeasible regions may reduce the dimensionality of the PF in this situation.

From the above analysis, it can be observed that PPS can deal with CMOP situations with all types of interactions among constraints and the unconstrained PF.

The main steps of PPS include the push and pull search processes. However, the decision as to when to switch from the push to the pull search process is very critical. A strategy for when to switch the search behavior is suggested as follows.

$$r_k \equiv \max \{r_{2k}, r_{nk}\} \leq \epsilon$$  \hspace{1cm} (4)

where $r_k$ represents the max rate of change between the ideal and nadir points during the last $l$ generations. $\epsilon$ is a user-defined parameter; for the examples in this paper, we have set $\epsilon = 1e^{-3}$. The rates of change of the ideal and nadir points during the last $l$ generations are defined in Eq. (5) and Eq. (6), respectively.

$$r_{2k} = \max \{ |z_k^i - z_k^{i-1}| \over \max |z_k^i|, \Delta \}$$  \hspace{1cm} (5)

$$r_{nk} = \max \{ |n_k^i - n_k^{i-1}| \over \max |n_k^i|, \Delta \}$$  \hspace{1cm} (6)

where $z_k = (z_1^k, \ldots, z_m^k)$, $n_k = (n_1^k, \ldots, n_m^k)$ are the ideal and nadir points in the $k$-th generation, and $z_k^i = \min_{j=1,...,N} f_j(x^j)$, $n_k^i = \max_{j=1,...,N} f_j(x^j)$. $N$ is the population size. $z_k^i - z_k^{i-1} = (z_1^k - z_1^{k-1}, \ldots, z_m^k - z_m^{k-1})$ are the ideal and nadir points in the $(k - 1)$-th generation. $r_{2k}$ and $r_{nk}$ are two points in the interval $[0, 1]$. $\Delta$ is a very small positive number, which is used to make sure that the denominators in Eq. (5) and Eq. (6) are not equal to zero. In this paper, $\Delta$ is set to $1e^{-6}$.

At the beginning of the search, $r_k$ is initialized to 1.0. At each generation, $r_k$ is updated according to Eq. (4). If $r_k$ is less than or equal to the predefined threshold $\epsilon$, the search behavior is switched to the pull search.

To summarize, PPS divides its search process into two different stages: push search and pull search. During the first stage, push search, which disregards the constraints, is adopted to approximate the unconstrained PF. Once Eq. (4) is satisfied, pull search is used to pull any infeasible solutions to the feasible and non-dominated regions—constraints are fully considered. PPS terminates when a predefined halting condition is met. In the following section, we will describe the instantiation of the push and pull strategy in a MOEA/D framework in detail.

### 3. An Instantiation of PPS in MOEA/D

This section describes the details of an instantiation of the push search method and the pull search method in the framework of a particular type of MOEA/D search, thus capturing the entire PPS method.

#### 3.1. The push search

In the push search stage, an unconstrained MOEA/D is used to search for non-dominated solutions without considering any constraints. When solving a MOP by using MOEA/D, we decompose the MOP into a set of single optimization subproblems and optimize them simultaneously in a collaborative way. Each subproblem is associated with a decomposition function by using a weight vector $\lambda$. In the decomposition-based selection approach, an individual is selected for survival into next generation based on the value of the decomposition function.

There are three popular decomposition approaches, including weighted sum [23], Tchebycheff [23] and boundary intersection approaches [26]. In this paper, we adopt the Tchebycheff decomposition method, with the detailed definition given...
Figure 1: Infeasible regions block the way towards the PF, and the unconstrained PF is the same as the constrained PF. (a)-(e) show the various stages of the push search process, and show the working population getting across the infeasible regions without any extra efforts dealing with constraints. (f) shows the pull search process, which is the same as (e) in this particular case, since the true PF is the same as the unconstrained PF, and has already been achieved by the working population in the push search process.

Figure 2: The unconstrained PF is covered by infeasible regions and all of it is infeasible. The true PF thus lies on some constraint boundaries. (a)-(c) show the push search process, in which the working population crosses the infeasible regions without any barriers. (d)-(f) show the pull search process, in which the infeasible solutions in the working population are gradually pulled to the feasible and non-dominated regions.
as follows.

\[ g^c(x, \lambda^i, z^*) = \max_{j=1,...,m} \frac{1}{\lambda_j^i} (f_j(x) - z_j^*) \]  

(7)

where \( \lambda^i \) is a weight vector, and \( \sum_{j=1,...,m} \lambda_j^i = 1, \lambda_j^i \geq 0 \). \( z^* \) is the ideal point, and \( z_j^* = \min_{k=1,...,N} f_j(x^k) \).

In the push search stage, a newly generated solution \( x \) is retained into the next generation based on the value of \( g^c \) as described in Algorithm 1.

Algorithm 1: Push Subproblem

1 Function result = PushSubproblems(x', y', z*)
2 result = false
3 if \( g^c(y', \lambda', z^*) \leq g^c(x', \lambda', z^*) \) then
4 \( x' = y' \)
5 result = true
6 end
7 return result
8 end

3.2. The pull search

In this process, infeasible solutions are pulled to the feasible and non-dominated regions. To achieve this, a constraint-handling mechanism is adopted to punish the infeasible solutions in the pull search stage. An improved epsilon constraint-handling method is proposed, with the detailed formulation given as follows.

\[ \epsilon(k) = \begin{cases} (1 - \tau) \epsilon(k-1), & \text{if } r_{f_k} < \alpha \\ \epsilon(0)(1 - \frac{1}{\epsilon})^p, & \text{if } r_{f_k} \geq \alpha \end{cases} \]  

(8)

where \( r_{f_k} \) is the ratio of feasible to infeasible solutions in the \( k \)-th generation. \( \tau \) is the parameter to control the speed of reducing the relaxation of constraints in the case of \( r_{f_k} < \alpha \), and \( \tau \in [0, 1] \). \( \alpha \) is to control the searching preference between the feasible and infeasible regions, and \( \alpha \in [0, 1] \). \( cp \) is to control the speed of reducing relaxation of constraints in the case of \( r_{f_k} \geq \alpha \). \( \epsilon(k) \) is updated until the generation counter \( k \) reaches the control generation \( T_c \). \( \epsilon(0) \) is set to the maximum overall constraint violation of the working population at the end of the push search. Compared with the \( \epsilon \) setting in [27], the proposed method in Eq. (8) has an exponential decreasing speed to set \( \epsilon(k) \) in the case of \( r_{f_k} < \alpha \), which can help to find feasible solutions more quickly and efficiently. In the case of \( r_{f_k} \geq \alpha \), the Eq. (8) is the same as the \( \epsilon \) setting in [27].

In the pull stage, a newly generated solution \( x \) is selected for survival into the next generation based on the value of \( g^c \), the overall constraint violation \( \phi(x) \) and the value of \( \epsilon(k) \), as illustrated by Algorithm 2. In the case of \( \epsilon(k) = 0 \), it is the same as the constraint-handling method proposed for MOEA/D in [28].

3.3. PPS Embedded in MOEA/D

Algorithm 3 outlines the pseudocode of PPS-MOEA/D. A CMOP is decomposed into \( N \) single-objective subproblems, and these subproblems are initialized at line 1. The ideal point \( z^* \) and the generation counter \( k \) are also initialized at line 1.
At line 2, \( r_k \), which is the maximum rate of change of ideal and nadir points, is initialized to 1.0, the flag of search stage is set to \( \text{true} \), the maximum overall constraint violation found so far is initialized to -1 (\( \text{maxViolation} = -1 \)).

The ideal and nadir points at \( k \)-th generation are set at line 3, and the details can be found in Algorithm 4. At line 5, the setting method can be found in Algorithm 5.

Then, the algorithm repeats lines 4-38 until \( T_{\text{max}} \) generations have been reached. The value of \( \varepsilon (k) \) and the search strategy are set at lines 5-16 based on Eq. (4) and Eq. (8). At line 5, \( \theta (k) \), the maximum rate of change of ideal and nadir points \( r_k \) is calculated, and the details can be found in Algorithm 6. \( \varepsilon (k) \) and \( \varepsilon (0) \) are initialized at line 9. Lines 18-34 show the process of updating subproblems. At line 18, \( S \) represents the neighbor indexes of solution \( x \). Lines 19-20 perform a DE operator to generate a new solution \( y \). Line 21 takes a polynomial mutation on \( y \), and generates a new solution \( y' \). Then the new solution \( y' \) is repaired as follows: If an element of \( y' \) is less than its lower boundary, it is reset to its lower boundary. If an element of \( y' \) is greater than its upper boundary, it is reset to its upper boundary.

The ideal point \( z^* \) is updated at line 22, and the details can be found in Algorithm 7. At line 23, the maximum overall constraint violation—\( \text{maxViolation} \) is updated.

From line 27 to 31, it can be seen that different search strategies are used to update subproblems. When \( \text{PushStage} = \text{true} \), the push search is adopted (line 28); otherwise, the pull search is used (line 30). At line 36, the generation counter \( k \) is updated. The ideal and nadir points at \( k \)-the generation are also set at this line, and the setting method can be found in Algorithm 4. Finally, the set of feasible and non-dominated solutions \( S_N \) is updated according to the non-dominated ranking as given in Algorithm 10 at line 37. The updating method can be found in Algorithm 8.

Algorithm 9 shows the process to set ideal and nadir points at \( k \)-th generation. At lines 2-5, the ideal and nadir points \( z^k \), \( n^k \) are initialized. Lines 9-11 update each element of the ideal point \( z^k \) and lines 12-14 update each element of the nadir point \( n^k \).

Algorithm 10 shows the process to update the maximum overall constraint violation found so far. The updating process is performed at lines 2-4. If the newly generated solution \( y' \) has a larger overall constraint violation (\( \phi (y') \)) than \( \text{maxViolation} \), \( \text{maxViolation} \) is set to \( \phi (y') \).

Algorithm 11 shows the pseudocode of calculating the maximum rate of change of ideal and nadir points \( r_k \). At lines 2-3, \( r_{z_k} \) and \( r_{m_k} \) are calculated according to Eq. 5 and Eq. 6, respectively. At lines 4, \( r_k \) is calculated according to Eq. 4.

Algorithm 12 shows the pseudocode of updating the ideal point \( z^* \). If the \( j \)-th objective of the newly generated solution \( y' \) has a smaller value (\( f_j(y') \)) than \( z^*_j \), then \( z^*_j \) is set to \( f_j(y') \).

Algorithm 13 shows the pseudocode of selecting feasible and non-dominated solutions. At lines 2-3, the number of feasible solutions (\( N_f \)) and the set of feasible solutions (\( \{ \bar{f}_{l,k} \} \)) are calculated. If \( N_f \) is smaller than the population size (\( N \)), the result is set to \( \{ \bar{f}_{l,k} \} \), as shown at lines 4-5. Line 7 performs non-dominated ranking on \( \{ \bar{f}_{l,k} \} \), and solutions in \( \{ \bar{f}_{l,k} \} \) are classified into \( q \) different fronts. Lines 9-12 add the first \( k-1 \) fronts to the result until the size of result is greater than \( (N - \{ \bar{f}_{l,k} \}).size() \). Lines 13-16 select the \( N \) result.size() solutions from \( k \)-th front (\( \{ \bar{f}_{l,k} \} \)) according to the crowding distance. The more details can be found in [19].

Remark: The proposed PPS is a general framework for solving CMOPs. Even though only PPS-MOEA/D is realized in this paper, it can be instantiated in many different MOEAs. At each search stage, a large variety of information can be gathered to extract useful knowledge that can be used to guide both search stages. In fact, knowledge discovery can be a critical step in the PPS framework. In this paper, we only utilize some statistical information. For example, the maximum overall constraint violation at the end of the push search stage is adopted to set the value of \( \varepsilon (0) \). The ratio of feasible to infeasible solutions at the pull search stage is used to control the value of \( \varepsilon (k) \). But in fact, many data mining methods and machine learning approaches can be integrated into the PPS framework for solving CMOPs more effectively and efficiently.

4. Experimental Study

4.1. Experimental Settings

To evaluate the performance of the proposed PPS method, six other CMOEAs, including MOEA/D-Epsilon [21], MOEA/D-Epsilon [22], MOEA/D-SR [23], MOEA/D-CDP [24], CMOEA/D [24] and NSGA-II-CDP [10], are tested on LIR-CMOP1-14 [21], which have large infeasible regions in the search space. The detailed parameters in each algorithm are listed as follows:

1. The mutation probability \( Pm = 1/n \) (\( n \) denotes the dimension of a decision vector). The distribution index in the polynomial mutation is set to 20.

2. DE parameters: \( CR = 1.0, f = 0.5 \).

3. Population size: \( N = 300 \). Neighborhood size: \( T = 30 \).

4. Halting condition: each algorithm runs for 30 times independently, and stops when 300,000 function evaluations are reached.
Algorithm 3: PPS-MOEA/D

**Input:** $N$: the number of subproblems, $T_{\text{max}}$: the maximum generation, $N$: Weight vectors: $\lambda^1, \ldots, \lambda^N$. $T$: the size of the neighborhood, $\delta$: the probability of selection from neighbors, $n_c$: the maximal number of solutions replaced by a child, $T_c$: the control generation for $s(k)$.

**Output:** $N$: a set of feasible non-dominated solutions

1. Decompose a CMOP into $N$ subproblems associated with weight vectors. Generate a population $P = \{x^1, \ldots, x^N\}$ randomly. For each $i = 1, \ldots, N$, set $B(i) = \{i_1, \ldots, i_T\}$, where $x^{i_1}, \ldots, x^{i_T}$ are the $T$ closest weight vectors to $x^i$.
2. Set the ideal point $z^i_j = \min_{i=1, \ldots, N} f_j(x^i)$. Set $k = 1$.
3. Set $r_k = 1.0$. $\text{PushStage} = \text{true}$, $\text{maxViolation} = -1$.
4. $\text{SetIdealNadirPoints}(P,k)$: For each $i = 1, \ldots, N$, $\text{UpdateMaxViolation}(x^i, \text{maxViolation})$;
5. while $k < T_{\text{max}}$ do
6.  if $k >= l$ then $\text{Set } r_k = \text{CalcMaxChange}(k)$ ;
7.  if $k < T$, then
8.    if $r_k \leq \epsilon$ and $\text{PushStage} == \text{true}$ then
9.      $\text{PushStage} = \text{false}$;
10.     $\epsilon(k) = \epsilon(0) = \text{maxViolation}$;
11.    end
12.   else
13.     $\text{Update } \epsilon(k)$ according to Eq. (8);
14.   end
15.  end
16.  for $i = 1$ to $N$ do
17.    if $\forall j < \delta$ then $S = B(i)$ else $S = \{1, \ldots, N\}$;
18.     Set $r^1 = i$ and select two indexes $r^2$ and $r^3$ from $S$ randomly, and $r^2 \neq r^3$.
19.     Generate a new solution $y$ from $x^{r^1}, x^{r^2}$ and $x^{r^3}$ by a DE operator.
20.     Perform a polynomial mutation on $y$ to generate a new solution $y^*$, and repair $y^*$.
21.     $\text{UpdateIdealPoint}(y^*, z^*)$;
22.     $\text{UpdateMaxViolation}(y^*, \text{maxViolation})$;
23.     Set $c = 0$.
24. while $c \neq n_c$ or $S \neq \emptyset$ do
25.  select an element $j$ from $S$ randomly.
26.  if $\text{PushStage} == \text{true}$ then
27.    result = $\text{PushSubproblems}(x^i, y^*, z^*)$;
28.  else
29.    result = $\text{PullSubproblems}(x^i, y^*, \epsilon(k), z^*)$;
30.  end
31.  if result == true then $c = c + 1$;
32.  $S = S \setminus \{j\}$;
33. end
34. $k = k + 1$; $\text{SetIdealNadirPoints}(P,k)$;
35. $N_S = \text{NDSel}(N_S \cup P, N)$;
36. end

Algorithm 4: Set Ideal and Nadir Points at $k$-th Generation

1. **Function** $\text{SetIdealNadirPoints}(P,k)$
2.  $\text{for } j = 1 \text{ to } m$ do
3.    $\text{// }$ is the number of objective.
4.    $z^i_j = 1e30$; $n^i_j = -1e30$;
5.  end
6.  $\text{for } i = 1 \text{ to } N$ do
7.    $\text{// } P(i)$ is the $i$-th solution in the population $P$
8.    if $f_j(P(i)) < z^i_j$ then
9.      $z^i_j = f_j(P(i))$
10.    end
11.  end
12. end

Algorithm 5: Update the Maximum Constraint Violation

1. **Function** $\text{UpdateMaxViolation}(y^*, \text{maxViolation})$
2.  if $\phi(y^*) > \text{maxViolation}$ then
3.    $\text{maxViolation} = \phi(y^*)$ // according to Eq. (3);
4. end

Algorithm 6: Calculate the Maximum Rate of Change of Ideal and Nadir Points $r_k$

1. **Function** $r_k = \text{CalcMaxChange}(k)$
2.  Calculate $r_{zk}$ according to Eq. (5).
3.  Calculate $rn_k$ according to Eq. (6).
4.  $r_k = \max(r_{zk}, rn_k)$, according to Eq. (4).
5. end

Algorithm 7: Update the Ideal Point

1. **Function** $\text{UpdateIdealPoint}(y^*, z^*)$
2.  $\text{for } j = 1 \text{ to } m$ do
3.    $\text{// } m$ is the number of objective.
4.    if $f_j(y^*) < z^i_j$ then
5.      $z^i_j = f_j(y^*)$
6.    end
7.  end
8. end
Algorithm 8: Select Feasible and Nondominated Solutions

Function result = NDSelect(\(\bar{P}, N\))
1. Calculate the number of feasible solutions \(N_{fs}\) in \(\bar{P}\).
2. Select a set of feasible solutions \(\bar{P}_{fs}\) from \(\bar{P}\).
3. if \(N_{fs} \leq N\) then
   4. \(result = \bar{P}_{fs}\)
   5. else
      6. Perform non-dominated ranking on \(\bar{P}_{fs}\), and generate \(q\) fronts according to \([10]\).
      7. Set \(k = 1\);
      8. while \((result.size() + \bar{P}_{fs}[k].size() < N)\) do
         9. \(result = result \cup \bar{P}_{fs}[k];\)
         10. \(k = k + 1;\)
      end
   end
   11. \(remainNo = N - result.size();\)
   12. Calculate the crowding distance \([10]\) for each solution in \(\bar{P}_{fs}[k];\)
   13. Select top \(remainNo\) solutions with the maximum crowding distance from \(\bar{P}_{fs}[k]\), denoted as \(R_{solutions}\).
   14. \(result = result \cup R_{solutions};\)
end
return result

5. Probability of selecting individuals from its neighborhood: \(\delta = 0.9\).
6. The max number of solutions updated by a child: \(nr = 2\).
7. Parameter setting in PPS-MOEAD: \(T_c = 800, \alpha = 0.95, c_p = 2, l = 20.\)
8. Parameter setting in MOEA/D-IEpsilon: \(T_c = 800, \alpha = 0.95, \tau = 0.1, \theta = 0.05N.\)
9. Parameter setting in MOEA/D-Epsilon: \(T_c = 800, c_p = 2, \alpha = 0.95, \tau = 0.1, \theta = 0.05N.\)
10. Parameter setting in MOEA/D-SR: \(S_c = 0.05.\)

4.2 Performance Metric

To measure the performance of PPS-MOEAD, MOEA/D-IEpsilon, MOEA/D-Epsilon, MOEA/D-SR, C-MOEAD, and NSGA-II-CDP, \([10]\), two popular metrics—the inverted generation distance (IGD) \([30]\) and the hypervolume \([31]\) are adopted.

- Inverted Generation Distance (IGD):

The IGD metric reflects the performance regarding convergence and diversity simultaneously. The detailed definition is given as follows:

\[
IGD(P^*, A) = \frac{1}{|P^*|} \sum_{y \in A} d(y^*, P^*)
\]

\[
d(y^*, A) = \min_{y \in A} \sqrt{\sum_{i=1}^{m} (y_i^* - y_i)^2}
\]

where \(P^*\) denotes a set of representative solutions in the true PF, and \(A\) is an approximate PF achieved by a CMOEA. \(m\) denotes the number of objectives. For two-objective LIR-CMOPs with continuous PFs, 1000 points are sampled uniformly from the true PF to construct \(P^*\). For LIR-CMOPs with three objectives, 10000 points are sampled uniformly from the true PF to constitute \(P^*\). It is worth noting that a smaller value of IGD may indicate better performance with regards to diversity and/or convergence. (Note that this measure cannot be used if the true PF is unknown, so it is used primarily for benchmarking purposes.)

- Hypervolume (HV):

HV reflects the closeness of the set of non-dominated solutions achieved by a CMOEA to the true PF. The largerHV means that the corresponding non-dominated set is closer to the true PF.

\[
HV(S) = \text{VOL}\left(\bigcup_{y \in S} [f_1(y), z_1^*] \times \ldots \times [f_m(y), z_m^*]\right)
\]

where \(\text{VOL}(\cdot)\) is the Lebesgue measure, \(m\) denotes the number of objectives, \(z^* = (z_1^*, \ldots, z_m^*)\) is a user-defined reference point in the objective space. For each LIR-CMOP, the reference point is placed at 1.2 times the distance to the nadir point of the true PF (Note that this particular placement of the reference point also requires knowledge of the true PF). A larger value of HV may indicate better performance regarding diversity and/or convergence.

4.3. Discussion of Experiments

4.3.1. Comparisons among PPS-MOEAD and the other six CMOEAs

The statistical results of the IGD values on LIR-CMOP1-14 achieved by PPS-MOEAD and the other six CMOEAs in 30 independent runs are listed in Table 1. According to the Wilcoxon-Test in this table, it is clear that PPS-MOEAD is significantly better than MOEA/D-Epsilon, MOEA/D-CDP, MOEA/D-SR, C-MOEAD and NSGA-II-CDP on all of the fourteen tested problems in terms of the IGD metric. For LIR-CMOP1, LIR-CMOP4, LIR-CMOP9-10, and LIR-CMOP13, there are no statistically significant differences between PPS-MOEAD and MOEA/D-I-Epsilon. For the rest of nine test problems, PPS-MOEAD is significantly better than MOEA/D-I-Epsilon. The statistical results of the HV values on LIR-CMOP1-14 achieved by PPS-MOEAD and the other six CMOEAs in 30 independent runs are listed in Table 2. It can be observed that PPS-MOEAD is significantly better than MOEA/D-Epsilon, MOEA/D-CDP, MOEA/D-SR, C-MOEAD and NSGA-II-CDP on all the test instances in terms of the HV metric. For LIR-CMOP4, LIR-CMOP9-10, and LIR-CMOP13, there are no statistically significant differences between PPS-MOEAD and MOEA/D-I-Epsilon. For the rest of ten test instances, PPS-MOEAD performs significantly better on these test problems. From the above observations, it is clear that the proposed PPS-MOEAD achieves significantly better
The three typical situations of infeasible regions influencing PFs as discussed in Section 2. In particular, for LIR-CMOP6, there are two large infeasible regions in front of the PF, and the unconstrained PF is the same as the constrained PF, as illustrated by Fig. 4(a). In Fig. 4(b), we can observe that PPS-MOEA/D and MOEA/D-IEpsilon can get across the large infeasible regions, while the rest of CMOEAs, including MOEA/D-Epsilon, MOEA/D-CDP, MOEA/D-SR, C-MOEA/D and NSGA-II-CDP are trapped in the boundary of infeasible regions as shown in Fig. 4(c)-(g). The reason is constraints are ignored in PPS-MOEA/D at the push stage, and MOEA/D-IEpsilon uses the improved epsilon-constraint-handling method to cross the infeasible regions (by dynamically adjusting the epsilon level to allow infeasible solutions to enter the population). As a result, the two large infeasible regions can not block the populations of PPS-MOEA/D/D and MOEA/D-IEpsilon to converge. However, the rest of five CMOEAs have no special mechanisms to cross large infeasible regions as illustrated in Fig. 4(a). As a result, the two large infeasible regions hinder their populations to converge. Therefore, they are trapped in the boundary of infeasible regions, which can be clearly observed in Fig. 4(c)-(g).

For LIR-CMOP7, there are three large infeasible regions, and the unconstrained PF is covered by infeasible regions and be no more feasible, as illustrated by Fig. 4(b). In Fig. 4(b), we can observe that PPS-MOEA/D and MOEA/D-IEpsilon can converge to the true PF, while the rest of five CMOEAs cannot converge to the true PF as shown in Fig. 4(c)-(g). The reason is that the three large infeasible regions can not hinder the populations of PPS-MOEA/D and MOEA/D-IEpsilon to converge. For PPS-MOEA/D, the unconstrained PF is first achieved at the push stage. Then, the population is pulled to the constrained PF by crossing only one infeasible region. Furthermore, the landscape of constraints has already been explored at the push process of PPS-MOEA/D. The maximum overall constraint violation can be calculated and applied to guide the $\epsilon$ parameter setting of the constraint-handling method in the pull stage. The reason is that MOEA/D-IEpsilon has no mechanisms to explore the landscape of constraints in advance, thus lacking the potential of setting the $\epsilon$ parameter properly, as what can be done in PPS-MOEA/D. For the other five CMOEAs, they have no special mechanisms to cross the large infeasible regions as illustrated in Fig. 4(b). As a result, the two large infeasible regions in front of the constrained PF hinder their populations to converge. Therefore, the proposed PPS-MOEA/D can find the true PF of LIR-CMOP7 reliably, and MOEA/D-IEpsilon can find the true PF of LIR-CMOP7 occasionally. However, the rest of five CMOEAs are trapped in the boundary of infeasible regions, which can be clearly observed in Fig. 4(c)-(g).

For LIR-CMOP11, infeasible regions make the original unconstrained PF partially feasible, as illustrated by Fig. 4(c). The PF of LIR-CMOP11 is disconnected and has seven Pareto optimal solutions, among which two of them are located on the unconstrained PF, and five are not. PPS-MOEA/D can find all the Pareto optimal solutions as shown in Fig. 4(a), while the other six CMOEAs can not find all the seven Pareto optimal solutions, as shown in Fig. 4(b)-(g). The reason is that at the push stage the infeasible regions present no barriers for the population of PPS-MOEA/D/D, and the unconstrained PF of LIR-CMOP11 can be obtained at the push stage. Since two Pareto optimal solutions are situated at the unconstrained PF, PPS-MOEA/D can find these two Pareto optimal solutions at the push stage instantly. Moreover, the landscape of constraints has already been explored at the push stage, which can help the searching of PPS-MOEA/D/D at the pull stage. The population of PPS-MOEA/D/D only needs to get cross one single infeasible region to find the other five Pareto optimal solutions at the pull stage. However, for the other six CMOEAs,
they need to get cross several infeasible and overlapping regions to find the Pareto optimal solutions. Besides, the landscape of constraints are not well explored in advance during the search, which makes it difficult for the other six CMOEAs to select constraint-handling parameters properly. As a result, the proposed PPS-MOEAD can find all the seven discrete Pareto optimal solutions reliably, while the other six CMOEAs can only find some Pareto optimal solutions occasionally.

According to the above observations and analysis, we can conclude that the proposed PPS-MOEAD performs significantly better than the other six CMOEAs on most test cases. The experimental results demonstrate that PPS-MOEAD can solve CMOPs well by taking advantage of both the push and pull strategies.

4.3.2. A Comparison between PPS-MOEAD and its Variant

By replacing the Eq. 8 with that of Takahama’s article for PPS-MOEAD, another version of PPS-MOEAD is obtained, denoted as PPS-MOEAD1. The same experimental parameters are adopted to test the performance of PPS-MOEAD1.

The statistical results of the IGD values on LIR-CMOP1-14 achieved by PPS-MOEAD1 and PPS-MOEAD in 30 independent runs are shown in Table 3. According to the Wilcoxon-Test in this table, it is clear that PPS-MOEAD1 is significantly better than PPS-MOEAD/D on LIR-CMOP1-4 and LIR-CMOP7-8. PPS-MOEAD1 is significantly better than PPS-MOEAD/D on LIR-CMOP5-6, LIR-CMOP9-10, and LIR-CMOP13. For LIR-CMOP11-12 and LIR-CMOP14, there are not significant differences between PPS-MOEAD/D and PPS-MOEAD1.

The statistical results of the HV values on LIR-CMOP1-14 achieved by PPS-MOEAD/D and PPS-MOEAD/D in 30 independent runs are listed in Table 4. It can be observed that PPS-MOEAD/D is significantly better than PPS-MOEAD/D on LIR-CMOP1-4, LIR-CMOP8, LIR-CMOP13-14, and it is significantly worse than PPS-MOEAD/D on LIR-CMOP5-6, LIR-CMOP9-10 and LIR-CMOP12. For the rest of test instances, there are not significant differences between PPS-MOEAD/D and PPS-MOEAD/D.

For LIR-CMOP1-4, PPS-MOEAD/D is significantly better than PPS-MOEAD/D in terms of both IGD and HV metrics. A common feature of these problems is that they have narrow feasible regions, and their constrained PFs are far away from their unconstrained PFs. At the push stage, PPS-MOEAD/D and PPS-MOEAD/D both can find the unconstrained PFs. At the pull stage, since all the solutions are infeasible, PPS-MOEAD/D adopts the first rule in Eq. (8) to decrease the epsilon values dramatically. Thus, PPS-MOEAD/D can quickly get across infeasible regions and find feasible solutions efficiently. In contrast, PPS-MOEAD/D decreases the epsilon value much more slowly as compared with PPS-MOEAD/D, which may slow down the process to get across infeasible regions to a large extent. When PPS-MOEAD/D find enough feasible solutions, it adopts the second rule in Eq. (8) to decrease the epsilon value slowly, which leads to a more thorough searching for feasible solutions. Therefore, PPS-MOEAD/D performs significantly better than PPS-MOEAD/D on LIR-CMOP1-4.

For LIR-CMOP5-6 and LIR-CMOP9-10, PPS-MOEAD/D performs significantly better than PPS-MOEAD/D. A common feature of LIR-CMOP5-6 and LIR-CMOP9-10 is that their constrained PFs are the same as their unconstrained counterparts, and located right on top of their unconstrained PFs. At the push stage, PPS-MOEAD/D and PPS-MOEAD/D can both find the unconstrained PFs. At the pull stage, PPS-MOEAD/D decreases the epsilon value much more slowly as compared with PPS-MOEAD/D, which lead to a more thorough searching for feasible solutions by using PPS-MOEAD/D. Therefore, for LIR-CMOP5-6 and LIR-CMOP9-10, PPS-MOEAD/D performs significantly better than PPS-MOEAD/D.

From the above observations and analysis, it is clear that the proposed PPS-MOEAD/D performs better than PPS-MOEAD/D in more cases. PPS-MOEAD/D is more suitable for solving CMOPs with constrained PFs located far away from their unconstrained PFs as illustrated in Fig. 2(a), while PPS-MOEAD/D is more suitable for solving CMOPs whose constrained PFs are located right on top of their unconstrained PFs as illustrated in Fig. 1(a).

4.3.3. The weakness of PPS-MOEAD

As illustrated in Section 4.3.1 PPS-MOEAD/D can be classified into two different search stages, including the push search and pull search. If the population of PPS-MOEAD/D is converged to the unconstrained PF with only one point, it is difficult to pull the population to the constrained PF. Base on the above hypothesis, we design a CMOP named TNK-v1 which is based on TNK [22] whose unconstrained PF only has one point. The detail definition of TNK-v1 is given as follows:

\[
\begin{align*}
    f_1(x) &= x_1 + g_1(x) \\
    f_2(x) &= x_2 + g_2(x) \\
    g_1(x) &= \sum_{j=1}^{J} (x_j - \sin(0.5x_j))^2 \\
    g_2(x) &= \sum_{j=1}^{J} (x_j - \cos(0.5x_j))^2 \\
    J_1 &= \{3, 5, \ldots, 29\}, J_2 = \{4, 6, \ldots, 30\} \\
    c_1(x) &= x_1^2 + x_2^2 - 1.0 - 0.1 \cos(16.0 \arctan\frac{x_1}{x_2}) \geq 0 \\
    c_2(x) &= 0.5 - (x_1 - 0.5)^2 - (x_2 - 0.5)^2 \geq 0 \\
    x_1, x_2 &\in [1e-4, \pi] \text{ and } x_3, x_4, \ldots, x_{30} \in [0, 1]
\end{align*}
\]

We use the same experimental parameters in Section 4.3.1 to test the seven CMOEAs on TNK-v1. The statistical results of the IGD and HV values on TNK-v1 achieved by each CMOEA are shown in Table 5. From this table, we can observe that PPS-MOEAD/D is significantly worse than the other six CMOEAs on TNK-v1. In Fig. 3 we can observe that PPS-MOEAD/D only find a part of the true PF. One possible reason is that, at the end of the push stage, the population of PPS-MOEAD/D is converged to a single unconstrained Pareto optimal point. The diversity of the population in PPS-MOEAD/D is lost, and it is difficult to pull the population to the whole true PF. From the experimental results, we can find that PPS-MOEAD/D is not suitable for solving CMOPs which have unconstrained PFs with only one point.
Table 1: IGD results of PPS and the other six CMOEAs on LIR-CMOP1-14. To facilitate the display of this table, PPS, Epsilon, Epsilon, CDP, and SR are short for MOEA/D-PPS, MOEA/D-Epsilon, MOEA/D-CDP, and MOEA/D-SR respectively. Wilcoxon's rank sum test at a 0.05 significance level is performed between PPS-MOEA/D and each of the other six CMOEAs. † and ‡ denote that the performance of the corresponding algorithm is significantly worse than or better than that of PPS-MOEA/D, respectively. '−D−' indicates PPS-MOEA/D is superior to, not significantly different from or inferior to the corresponding compared CMOEAs.

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<th>Epsilon</th>
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Wilcoxon-Test (S-D-I) – 9.5-0 14-0-0 14-0-0 14-0-0 14-0-0
Table 2: HV results of PPS-MOEA/D and the other six CMOEAs on LIR-CMOP1-14. To facilitate the display of this table, PPS, IEpsilon, Epsilon, CDP, and SR in this table are short for MOEA/D-PPS, MOEA/D-Epsilon, MOEA/D-Epsilon, MOEA/D-CDP, and MOEA/D-SR respectively. Wilcoxon’s rank sum test at a 0.05 significance level is performed between PPS-MOEA/D and each of the other six CMOEAs. † and ‡ denote that the performance of the corresponding algorithm is significantly worse than or better than that of PPS-MOEA/D, respectively. ‘S-D-I’ indicates PPS-MOEA/D is superior to, not significantly different from or inferior to, the corresponding compared CMOEAs.

<table>
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<th>CDP</th>
<th>SR</th>
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Figure 5: The non-dominated solutions achieved by each algorithm on LIR-CMOP6 with the median HV values.

Figure 6: The non-dominated solutions achieved by each algorithm on LIR-CMOP7 with the median HV values.
Figure 7: The non-dominated solutions achieved by each algorithm on LIR-CMOP11 with the median HV values.

Figure 8: The non-dominated solutions achieved by each algorithm on TNK-v1 with the median HV values.
Table 3: IGD results of PPS-MOEA/D and PPS-MOEA/D1 on LIR-CMOP1-14. Wilcoxon’s rank sum test at a 0.05 significance level is performed between PPS-MOEA/D and PPS-MOEA/D1. † and ‡ denote that the performance of MOEA/D1 is significantly worse than or better than that of PPS-MOEA/D, respectively. ‘S-D-I’ indicates PPS-MOEA/D is superior to, not significantly different from or inferior to PPS-MOEA/D1.

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Wilcoxon-Test (S-D-I) – 6-3-5

Table 4: HV results of PPS-MOEA/D and PPS-MOEA/D1 on LIR-CMOP1-14. Wilcoxon’s rank sum test at a 0.05 significance level is performed between PPS-MOEA/D and PPS-MOEA/D1. † and ‡ denote that the performance of MOEA/D1 is significantly worse than or better than that of PPS-MOEA/D, respectively. ‘S-D-I’ indicates PPS-MOEA/D is superior to, not significantly different from or inferior to PPS-MOEA/D1.

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Wilcoxon-Test (S-D-I) – 7-2-5
5. Robot Gripper Optimization

In this section, a real-world optimization problem—the robot gripper optimization problem is formulated. Then, the proposed PPS-MOEA/D and the other six CMOEAs are tested on this optimization problem.

### 5.1. The formulation of the robot gripper optimization problem

The robot gripper optimization problem has two objectives and eight constraints, which is taken from [33]. The second and the fourth objectives of the original problem are used to formulate the robot gripper optimization problem in this work, while the constraints and the ranges of decision variables are kept the same as those in [33]. In this paper, the first objective $f_1(x)$ represents a force transmission ratio between the actuating force and the minimum gripping force. We prefer to transform more actuating force into the gripper force. Thus, this objective should be minimized. The second objective $f_2(x)$ is the sum of all elements of the robot gripper. It is relevant to the weight of the robot gripper, and minimizing $f_2(x)$ can lead to a lightweight design.

To study the characteristics of the robot gripper optimization problem, 3,000,000 solutions are generated as shown in Fig. 9 where 1,500,000 solutions are generated randomly, and the other 1,500,000 solutions are generated by MOEA/D-Epsilon. We can observe that the two objectives are in conflict with each other.

### 5.2. Experimental study

#### 5.2.1. Experimental settings

Five CMOEAs, including PPS-MOEA/D, MOEA/D-I-Epsilon [21], MOEA/D-SR [23], MOEA/D-CDP [22], C-MOEA/D [24] and NSGA-II-CDP [10], are tested on the robot gripper optimization problem. The parameters of these five CMOEAs are the same as listed in Section 5.1 except for the termination conditions. In the robot gripper optimization problem, each CMOEA stops when 600,000 function evaluations are reached. Since the true PF of the robot gripper optimization problem is unknown, We use the hypervolume metric [31] to measure the performance of these five CMOEAs, and the reference point is set to $[5, 800]^7$.

#### 5.2.2. Analysis of experiments

The statistical results of $HV$ values achieved by PPS-MOEA/D and the other six CMOEAs are shown in Table 5. We can observe that PPS-MOEA/D is better or significantly better than the other six CMOEAs. The non-dominated solutions achieved by each CMOEA with the median $HV$ values during the 30 independent runs are plotted in Fig. 10(a)-(g). It is clear that PPS-MOEA/D has better or significantly better performance than the other six CMOEAs.

Three representative individuals (A, B and C) are selected from the non-dominated solutions achieved by PPS-MOEA/D with the best $HV$ value in the 30 independent runs. The configurations of the robot gripper mechanism at each point are plotted in Fig. 11. We can observe that $f_1(x)$ is increasing with the decreasing of $f_2(x)$ from A to C, and each individual has a different geometrical structure.

The non-dominated solutions achieved by each algorithm on TNK-v1 with the median $HV$ values.

### 6. Conclusion

This paper proposes a general PPS framework to deal with CMOPs. More specifically, the search process of PPS is divided into two stages—namely, push and pull search processes. At the push stage, constraints are ignored, which can help PPS to cross infeasible regions in front of the unconstrained PF. Moreover, the landscape affected by constraints can be estimated during the push stage, and this information, such as the ratio of feasible to infeasible solutions and the maximum overall constraint violation, can be applied to conduct the settings of parameters coming from the constraint-handling mechanisms in
Table 6: HV results of PPS-MOEA/D and the other six CMOEAs on the gripper optimization problem. To facilitate the display of this table, PPS, IEpsilon, Epsilon, CDP, and SR in this table are short for MOEA/D-PPS, MOEA/D-IEpsilon, MOEA/D-Epsilon, MOEA/D-CDP, and MOEA/D-SR respectively. Wilcoxon’s rank sum test at a 0.05 significance level is performed between PPS-MOEA/D and each of the other six CMOEAs. † and ‡ denote that the performance of the corresponding algorithm is significantly worse than or better than that of PPS-MOEA/D, respectively. The best mean is highlighted in boldface.

<table>
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<th>Epsilon</th>
<th>SR</th>
<th>CDP</th>
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<td>8.070E+00</td>
<td>9.435E+00</td>
<td>6.055E+01</td>
</tr>
</tbody>
</table>

Figure 10: The non-dominated solutions achieved by each algorithm on the robot gripper optimization problem with the median HV values are plotted in (a)-(g). The reference PF consists of the non-dominated solutions achieved by each algorithm.

Figure 11: The non-dominated solutions achieved by PPS-MOEA/D with the best HV value in the 30 independent runs.
the pull stage. When the max rate of change between ideal and
nadir points is less or equal than a predefined threshold, PPS
is switched to the pull search process. The infeasible solutions
achieved in the push stage are pulled to the feasible and non-
dominated area by adopting an improved epsilon constraint-
handling technique. The value of epsilon level can be set prop-
erly according to the maximum overall constraint violation ob-
tained at the end of the push search stage. The comprehensive
experiments indicate that the proposed PPS-MOEA/D achieves
significantly better results than the other six CMOEAs on most
of the benchmark problems and the robot gripper optimization
problem.

It is also worthwhile to point out that there has been very lit-
tle work regarding using information of landscape affected by
constraints to solve CMOPs. In this context, the proposed PPS
provides a viable framework. Obviously, a lot of work need to
be done to improve the performance of PPS, such as, the strat-
egy of searching around the borders between infeasible and fea-
sible regions, the augmented constraint-handling mechanisms
in the pull stage, the enhanced strategies to switch the search
behavior, and the data mining methods and machine learning
approaches integrated in the PPS framework. For another fu-
ture work, the proposed PPS will be implemented in the non-
dominated framework, such as NSGA-II, to further verify the
effect of PPS. More other CMOEAs and real-world optimization
problems will also be used to test the performance of the PPS
embedded in different MOEA frameworks.

Acknowledgement

This research work was supported by Guangdong Key Lab-
oratory of Digital Signal and Image Processing, the National Nat-
ural Science Foundation of China under Grant (61170737,
61300159, 61332002, 51375287), Jiangsu Natural Science	Foundation (BK20130808) and Project of Internation as well as
Hong Kong, Macao & Taiwan Science and Technology Co-
operation Innovation Platform in Universities in Guangdong

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