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Evolutionary Linear Control Strategies of Triple Inverted Pendulums and Simulation Studies

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***Abstract* - Research significance, development in theories and applications and several classical intelligent control strategies for inverted pendulum(s) are described briefly first. Evolutionary linear control strategy is presented to triple inverted pendulums, with the characteristics of complex non-linearity, strong duality and quick time-variance. Under the guidance of comprehensive performance function weighted ITAE, this strategy searches for the satisfactory parameters by adaptive genetic algorithm the authors presented where the crossover and mutation probability are changed with the fitness function value nonlinearly. Tracking simulations, anti-disturbance and robust experiments are given. The simulation results illustrate that this strategy has ideal dynamic, steady performance, anti-disturbance and robustness.**

Index Terms - Triple inverted pendulum. Adaptive genetic algorithm. Evolutionary linear control.

I. INTRODUCTION

Inverted pendulum, in particular multi-staged inverted pendulum, is a typical complicated, strongly coupled and unstable nonlinear system that defies precise modeling and control using quantitative methods. For example, to control a triple inverted pendulum, only one motor is used as actuator to control up to eight state variables. At the same time, inverted pendulum system are analogous and thus can approximately represent and emulate many real-life systems. Examples include 1) a rocket booster on lift-off can be simulated as a single link inverted pendulum mounted on a movable cart. 2) The study of a triple link system can be approximately simplified as a model of the human standing on one leg and thus help the understanding and development of biped locomotive machines [1]. As such, inverted pendulum systems have been extensively studied to test efficiency and effectiveness of various control algorithms and schemes, as well as a good tool to train control engineers and engineering students.

Due to its great value for research and train purposes, multi-staged inverted pendulum has been tackled by a large variety of approaches including classical control theories, advanced control algorithms and intelligent control schemes. During the last decades, some prominent progresses have been achieved. Furuta *et al* successfully controlled a double inverted pendulum using state space linear control algorithm in 1978 [2]. In 1984, Furuta also successfully controlled a triple inverted pendulum using two motors—however in his control system, there was no moving cart taken into account and the first pendulum was fixed to the ground [3]; Eltohamy who is now working for Honeywell investigated triple inverted pendulum during his Ph. D program at University of Arizona and developed a nonlinear optimal control algorithm for triple inverted pendulum using only one single motor [1,4]; Recently Petko Petkov's group reported their research on using robust control algorithm to control triple inverted pendulum. They constructed a low order controller that has expected robust stability and robustness using mu-analysis and SLICOT [5]; The researchers in the just organized Max-Planck complexity technology system laboratory have made significant contributions to inverted pendulum control and have successfully controlled triple inverted pendulum; Zhang, et al in Beijing University of Aeronautics and Astronautics has long been investigating inverted pendulum. He implemented intelligent control scheme to control triple inverted pendulum in both horizontal and skew rail tracks [6]; Shen and Zhang et al have studied the controllability, observability and stability of triple inverted pendulum system and realized humanoid digital control of triple inverted pendulum; Li et al effectively controlled inverted pendulum system using the Cloud Model they developed. Their paper " A Novel Qualitative Control Method to Inverted Pendulum Systems " has won the best paper award in the IFAC'99. Recently Li et al has successfully controlled four-staged inverted pendulum, the most difficult type of inverted pendulum control problem up to by now [9].

In this paper, an effective evolutionary linear control strategy is presented for the control of triple inverted pendulum system. The latter part is organized as follows: Section 2 gives the relative models and symbols; Section 3 describes the control strategy; Section 4 introduces the simulated results; Section 5 concludes the full paper and simply states another novel control approach.

II. MODELS AND SYMBOLS

A Mathematical Model of Multi-Staged Inverted Pendulum

$$\ddot{q}=H^{-1}(\tau-h\dot{q}-G) \tag{1}$$

$$q=[r,\theta_1,\theta_2,\theta_3]'$$

$$\tau=\begin{bmatrix}F\\0\\0\\0\end{bmatrix} \qquad G=\begin{bmatrix}0\\-[m_1l_1+(m_2+m_3)L_1]g\sin\theta_1\\-(m_2l_2+m_3L_2)g\sin\theta_2\\-m_3l_3g\sin\theta_3\end{bmatrix} \tag{2}$$

$$H=\begin{bmatrix}M+m_1+m_2+m_3 & [m_1l_1+(m_2+m_3)L_1]\cos\theta_1 & (m_2l_2+m_3L_2)\cos\theta_2 & m_3l_3\cos\theta_3\\ [m_1l_1+(m_2+m_3)L_1]\cos\theta_1 & m_1l_1^2+(m_2+m_3)L_1^2+J_1 & (m_2l_2+m_3L_2)L_1\cos(\theta_1-\theta_2) & m_3L_1l_3\cos(\theta_1-\theta_3)\\ (m_2l_2+m_3l_2)\cos\theta_2 & (m_2l_2+m_3L_2)L_1\cos(\theta_1-\theta_2) & m_2l_2^2+m3L_2^2+J_2 & m_3L_2l_3\cos(\theta_2-\theta_3)\\ m_3l_3\cos\theta_3 & m_3L_1l_3\cos(\theta_1-\theta_3) & m3L2l3\cos(\theta_2-\theta_3) & m_3l_3^2+J_3\end{bmatrix} \tag{3}$$

$$h=\begin{bmatrix}0 & -[m_1l_1+(m_2+m_3)L_1]\theta_1\sin\theta_1 & -(m_2l_2+m_3L_2)\theta_2\sin\theta_2 & -m_3l_3\theta_2\sin\theta_3\\ 0 & 0 & (m_2l_2+m_3L_2)L_1\theta\theta_2 & m_3L_1l_3\theta_3\sin(\theta_1-\theta_3)\\ 0 & -(m_2l_2+m_3L_2)L_1\theta_1\sin(\theta_1-\theta_2) & 0 & m_3L_2l_3\theta_3\sin(\theta_2-\theta_3)\\ 0 & -m_3L_1l_3\theta_1\sin(\theta_1-\theta_3) & -m_3L_2l_3\theta_2\sin(\theta_2-\theta_3) & 0\end{bmatrix} \tag{4}$$

Model and symbol descriptions of triple inverted pendulum is as following:

TABLE I

SYMBOLS FOR THE MODEL OF TRIPLE INVERTED PENDULUM

parameters	value	significance	parameters	value	significance
M	1.3280	Mass of the cart (kg)	J_3	0.0040	Mass moment of inertia of the upper swing/link about its center of gravity (kg . m²)
M_1	0.22	Mass of the lower swing/link(kg)	l_1	0.304	Distance from the lower position sensor to the center of gravity of the lower swing/link (m)
M_2	0.187	Mass of the middle swing/link (kg)	l_2	0.226	Distance from the middle position sensor to the center of gravity of the lower swing/link (m)
M_3	0.16	Mass of the upper swing/link (kg)	l_3	0.20	Distance from the upper position sensor to the center of gravity of the lower swing/link (m)
J_1	0.004963	Mass moment of inertia of the lower swing/link about its center of gravity (kg . m²)	L_1	0.49	Distance from the lower position sensor to the middle position sensor (m)
J_2	0.004824	Mass moment of inertia of the middle swing/link about its center of gravity (kg . m²)	L_2	0.45	Distance from the middle position sensor to the upper position sensor (m)

B Numerical computation for the model

To the differential equation: $y=f(x,y)$, the standard fouth-order Roung-Kutta recursive formula is as follows:

As an example, to the double pendulum, set: $x_1=r$; $x_2=\theta_1$; $x_3=\theta_2$; $x_4=\theta_3$; $x_5=r'$; $x_6=\theta'_1$; $x_7=\theta'_2$; $x_8=\theta'_3$ then the equation (1) can be written as:

$$\begin{cases} y_{n+1}=y_n+\frac{1}{6}(K_1+2K_2+2K_3+K_4)\\ K_1=hf(x_n,y_n)\\ K_2=hf(x_n+\frac{h}{2},y_n+\frac{1}{2}K_1)\\ K_3=hf(x_n+\frac{h}{2},y_n+\frac{1}{2}K_2)\\ K_4=hf(x_n+h,y_n+K_3)\end{cases} \tag{5}$$

$$\begin{bmatrix}\dot{x}_1\\\dot{x}_2\\\dot{x}_3\\\dot{x}_4\\\dot{x}_5\\\dot{x}_6\\\dot{x}_7\\\dot{x}_8\end{bmatrix}=\begin{bmatrix}x_5\\x_6\\x_7\\x_8\\ H^{-1}(x_2,x_3,x_4)(\tau-h\begin{bmatrix}x_5\\x_6\\x_7\\x_8\end{bmatrix}-G)\end{bmatrix} \tag{6}$$

The above equation is convenient for the computing by Roung-Kutta method.

III. EVOLUTIONARY LINEAR CONTROL STRATEGY

A Control algorithm and system diagram

Based upon the idea of humanoid intelligent control, the control strategy represents control variable with linear combination of state variables. Then algorithms of evolutionary computation are used to search for the optimal coefficient set to obtain the satisfactory global index function. Wherein linear control scheme is as follows

To the double pendulum

$$U=k_1x+k_2\dot{x}+k_3\theta_1+k_4\dot{\theta}_1+k_5\theta_2+k_6\dot{\theta}_2 \tag{7}$$

To the triple pendulum

$$U=k_1x+k_2\dot{x}+k_3\theta_1+k_4\dot{\theta}_1+k_5\theta_2+k_6\dot{\theta}_2+k_7\theta_3+k_8\dot{\theta}_3 \tag{8}$$

Structure of the triple inverted pendulum control system is shown in Fig. 1:

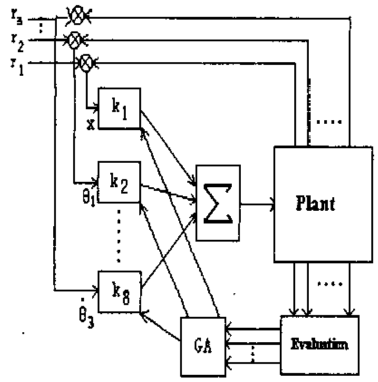

Fig. 1 System diagram

The reason that we can use linear control scheme to control such a complex system is that in most cases the initial state of the system is located around the equilibrium state.

B evolutionary optimization of the linear controller

The coefficients k_i (i=1, 2, ... , 8) are optimized by an adaptive genetic algorithm we developed. The algorithm has the following features: (1) Elitism and migration are used. In each generation, we select the best 2%~5% best individuals and take them directly into the next generation without crossover, mutation and reproduction operation. Meanwhile, we discard the worst 20%~40% individuals and relinquish them with randomly created new individuals. In this way, the evolutionary pool is open and can be refreshed by new individuals in each generation. This significantly enhances the possibility of finding global optimal solution. (2) The evolutionary operators can adapt themselves. The crossover rate Pc and mutation rate Pm are the most significant parameters to effect the performance of the genetic algorithm. However, Pc and Pm in most cased are determined by human subjectively. In our research, we make Pc and Pm adaptable, namely, for individuals with bigger fitness we set smaller values for Pc and Pm to protect them; and for individuals with smaller fitness we set bigger values for Pc and Pm to expedite their evolution. The equations are formulated as following:

$$P_c = \begin{cases} k_1 - k_2 \exp\{k_3(f - \overline{f})\} & f \geq \overline{f} \\ k_4 & \end{cases} \quad (9)$$

$$P_m = \begin{cases} k_1' - k_2' \exp\{k_3'(f - \overline{f})\} & f \geq \overline{f} \\ k_4' & \end{cases} \quad (10)$$

For simplicity, set k_3, k_3' to 1. We can derive the following:

$$k_1 = \frac{\exp\{f_{\max} - \overline{f}\}P_{c_1} - P_{c_0}}{\exp\{f_{\max} - \overline{f}\} - 1} \quad (11)$$

$$k_2 = \frac{P_{c_1} - P_{c_0}}{\exp\{f_{\max} - \overline{f}\} - 1} \quad (12)$$

$$k_1' = \frac{\exp\{f_{\max} - \overline{f}\}P_{m_1} - P_{m_0}}{\exp\{f_{\max} - \overline{f}\} - 1} \quad (13)$$

$$k_2' = \frac{P_{m_1} - P_{m_0}}{\exp\{f_{\max} - \overline{f}\} - 1} \quad (14)$$

In [11], more specific discuss where coefficients k_3, k_3' are arbitrary is given.

Definition of fitness function is based upon global performance criteria/index function ITAE (take the example of

$$ITAE = \int_0^t t \cdot E(t)dt \quad (15)$$

$$E(t) = \alpha_1|x| + \alpha_2|\theta_1| + \alpha_3|\theta_2| + \alpha_4|\theta_3| \quad (16)$$

triple inverted pendulum).Here, we adopted a weighted ITAE index function because of the consideration that the control

$$f(x) = c_1 + (c_2 - c_1)\frac{(ITAE)_{\max} - ITAE(x)}{(ITAE)_{\max} - (ITAE)_{\min}} \quad (17)$$

difficulty of upper, middle and lower swing/link decreases. $0 < \alpha_1 < \alpha_2 < \alpha_3 < \alpha_4$; α_1=0.2 ; α_2=0.4 ; α_3=0.6 ; α_4=0.8 ;

After this, fitness function is calculated according to the following formula:

Formula(17) maps the value of each seed's *ITAE* into the interval $[c_1,c_2] \subset [0,1]$ based on the principle: the less, the better. Where (*ITAE*) max 、(*ITAE*) min represents the maximum and minimum of the fitness in a generation respectively.

Roulette wheel mechanism is used for reproduction operation. Other related parameters include: population size 100~200, c_1=0.4, c_2=0.9; k_4=0.8; For simplicity, mutation rate in this research is always set to 0.1.

IV. SIMULATIONS

Sampling period: 0.02s; simulation time: 12s; initial conditions: $\theta_1 = \theta_2 = \theta_3$=0.03rad; other initial values of state variables are zeros.

1)the results without noise (Fig. 2)

ITAE=0.181 ;

K=[-14.0107,-1.64345,17.396,-17.9086,-25.5142,-1.73772,0.776265,-9.69736]

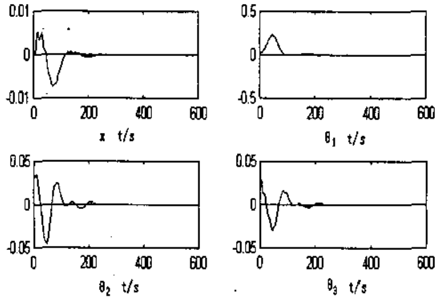

Fig. 2 The simulated curves without noise

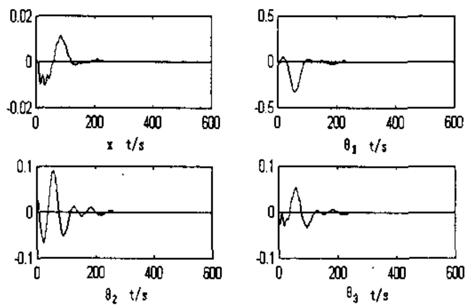

Fig. 3 The robustness curves

2)robustness experiments（Fig. 3）: The initial states is changed into:θ_1=-0.03rad, θ_2=0.03rad, θ_3=0.03rad (K is as above)

3)the results after adding noise in x and θ_1（Fig. 4）: (K is as above)

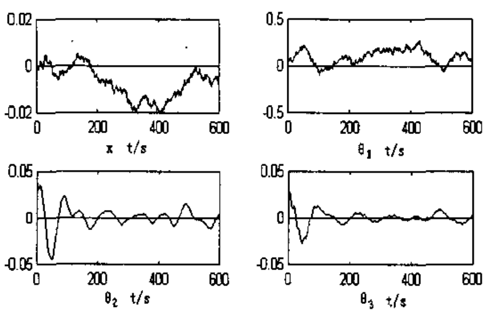

Fig. 4 The curves of disturbance-resistance

Where x is added noise $n \in$[-0.001m,0.001m]; θ_1 is added noise $n \in$ [-0.02rad,0.02rad].

From the experiment results we have the following conclusions:

1)The evolutionary control strategy presented in this paper has very good static and transient characteristics. Its smoothness is better than other published approaches up to date.

2)The evolutionary control strategy has enough robustness and can work well under noise environment.

3)The algorithm is computationally intensive. It is suitable to run the algorithm offline first and then fine-tuned online.

V. CONCLUSIONS

This paper presents an evolutionary linear control strategy to control a typical kind of multi-staged inverted pendulums-triple inverted pendulum. Simulation results show the robustness of the strategy as well as its good static and transient characteristics. In addition, according to the strong coupling of multiple links in the inverted pendulum, an evolutionary approach based on principal component analysis is studied as an extension to the strategy discussed in this paper. The detailed algorithms will be presented in another paper.

REFERENCES

[1] K.-G. Eltohamy, C.-Y. Kuo, "Nonlinear optimal control of a triple link inverted pendulum with single control input," *Int. J Control*, vol. 69, no. 2, pp. 239-256, 1998.

[2] K. Furuta, T. Okutani, and H. Sone, "Computer control of a double inverted pendulum," *Computer and Electrical Engineering*, no. 5, pp. 67-84, 1978.

[3] K. Furuta, T. Okutani, and N. Ono, "Attitude control of a triple inverted pendulum," *Int. J Control*, vol. 39, pp. 1351-1356, 1984.

[4] K.-G. Eltohamy, "Stabilization of a triple link inverted pendulum with single control input," PhD dissertation, Arizona State University, 1997.

[5] P. Petkov, "Robust control of a triple inverted pendulum using mu-synthesis," SLICOT Working Note 2001-1, http://www.mpimagdeburg.mpg.de/research/pendel/index_e.html

[6] Y.-W. Yang, M.-N. Zhang, "Stabilizing triple-inverted pendulum using computer," *Journal of Beijing University of Aeronautics and Astronautics*, vol. 26, no. 3, pp. 311-314, 2000.

[7] Y.-Y. He, C.-Z. Shen, "Controllability and observability of triple-inverted pendulum." *Journal of Beijing University of Aeronautics and Astronautics*, vol. 22, no. 5, pp. 545-549, 1996.

[8] F.-Z. Zhang, C.-Z. Shen, Y.-Z. Fan, "Imitating-human intelligent control of triple link inverted-pendulum," *Computer Engineering and Applications*, no. 2, pp. 17-20, 2000.

[9] H.-X. Li, Z.-H. Miao and J.-Y. Wang, "Variable universe self adaptive controller of fourth stage inverted pendulum," *Science in China* (Series E), vol. 32, no. 1, pp. 65-75, 2002.

[10]P. Wang, C.-Z. Xu *et al*, "A novel evolution-fuzzy control algorithm for complex systems," *Journal of Systems Engineering and Electronics.*, vol. 13, no. 3, pp. 52-60, 2002.

[11] P. Wang, "Applicational researches with soft computing for some decision and control issues," PhD dissertation, Huazhong University of Science and Technology, 2003.