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Fixed-Time Attitude Tracking Control for Rigid Spacecraft without Angular Velocity Measurements

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***Abstract*—This paper considers the problem of velocity-free fixed-time attitude tracking control for rigid spacecraft. With the help of the homogeneity theorem, a semi-global observer is introduced to estimate the unmeasured angular velocities within fixed time. Then, a velocity-free attitude tracking controller is designed to make the spacecraft attitude track a time-varying reference signal in finite time which can be up bounded by a fixed number regardless of the initial conditions. Finally, numerical examples are provided to illustrate the efficiency of the present control scheme.**

***Index Terms*—Attitude control, fixed-time control, homogeneity property, rigid spacecraft.**

I. INTRODUCTION

SPacecraft attitude control has been extensively studied in the past decades due to it's wide application in various space missions. The problem of attitude control of spacecraft has been well understood for the case when full attitude states (i.e. both spacecraft attitude and angular velocity) are measurable. However, in realistic applications, because of sensor failures and/or the cost reduction in on-board sensors, measurements of angular velocity might be not available for the controller development. It should be pointed out that the design of a velocity-free attitude control system could be challenging because of the unavailability of angular velocity measurements. In the literature, the problem of velocity-free attitude control for spacecraft has been investigated by using filters [1]–[3], auxiliary dynamical systems [4], reduced-order observers [5], and full-order observers [6]–[12] to remove the requirement of angular velocity measurements.

On the other hand, finite-time control which can guarantee finite-time convergence of the system trajectory to the equilibrium state has been an active research topic in the control community in the past few years as a finite-time control scheme can lead to higher accuracy control performance, stronger robustness against disturbances and a faster convergence rate (if the state of a dynamic system is near the equilibrium state) as compared with an asymptotic control law. In the literature, the finite-time attitude control has been investigated by using the terminal sliding mode approach [13]–[17], the method of "adding a power integrator" [18], and the homogeneity theorem [19], [20]. In [17], using the terminal sliding mode control and model predictive control, a double layer compound controller was designed for attitude control of rigid spacecraft. It is worth noting that full-state measurements are required for the implementation of the aforementioned finite-time attitude control schemes. Considering the unavailability of angular velocity measurements, several researchers [7]–[12] have developed full-order finite-time observers to estimate unmeasured angular velocities, and then designed velocity-free finite-time attitude controllers with application of the terminal sliding mode method [11], [12], the adding a power integrator technique [8] and the homogeneity property [7], [9], [10].

Finite-time control schemes may suffer from two drawbacks. The first drawback is that the finite-time controller has a slower convergence rate than an asymptotic controller if the system state is far away from the equilibrium state. The second one is that the settling time relies on the initial conditions heavily. One solution to overcome these two drawbacks is the fixed-time control [21]–[23]. The fixed-time control scheme can produce some required control precision within a given time independent of initial conditions [23]. By using the sliding mode control and polynomial feedbacks, Polyakov [22] designed a class of fixed-time controllers for stabilizing control of linear systems. Based on Implicit Lyapunov Function, Polyakov *et al*. [23] developed fixed-time control schemes for stabilizing control of a chain of integrators. Using the modified terminal sliding surface [24], nonsingular fixed-time-based sliding surfaces were proposed in [25] and [26] for spacecraft attitude control. In [27], an adaptive fixed-time terminal sliding mode attitude control law was proposed for rigid spacecraft. Recently, Sun *et al*. [28] designed a fixed-time attitude tracking

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control scheme by using the technique of adding a power integrator. However, it should be pointed out that such a technique is focused on dominating some nonlinearities of the system dynamics but not canceling them in the feedback design [29]. Thus, the control gains and consequently the applied control torques are usually required to be large to ensure the fixed-time convergence of the closed-loop system. In addition, full-state measurements are necessary for the implementation of the above fixed-time control schemes. In [30], a fixed-time control scheme was designed for output feedback control of double integrator systems. Due to the inherent nonlinearity of the spacecraft dynamics, the fixed-time output feedback control law developed in [30] is not applicable to solve the problem of attitude control of spacecraft. Recently, a fixed-time output feedback controller was proposed in [31] for a class of multiple-input multiple-output nonlinear systems under the globally Lipschitz assumption. However, the system dynamics is supposed to be exactly known and the effect of measurement noise is not examined.

Motivated by the above observations, this paper is devoted to studying the problem of fixed-time attitude tracking control for rigid spacecraft without angular velocity measurements. Using modified Rodrigues parameters (MRPs) as the attitude representation, a novel velocity-free fixed-time attitude tracking controller is developed for rigid spacecraft by use of the homogeneity property. The present control scheme is continuous and nonsingular. It should be emphasized that the stability results stated in the present work refer to the attitude system using the MRPs-based attitude parameterizations. The main contributions of the present paper are: (1) The convergence time of the proposed scheme is independent of initial conditions but the initial conditions may play a decisive role in the convergence time of the finite-time attitude controllers in [7]–[16], [18]–[20]; (2) Compared with the fixed-time attitude controllers presented in [25]–[28], the proposed fixed-time control scheme does not require angular velocity measurements, and thus it can reduce the cost of on-board sensors; (3) In contrast to the fixed-time output feedback control law in [30], the proposed scheme is applicable to solve the problem of fixed-time output feedback control of a class of second-order nonlinear systems; (4) In comparison with the fixed-time output feedback controller in [31], the stability analysis of the resulting closed-loop system before the convergence of the fixed-time observer is presented, the effect of uncertainty is investigated, and the effect of measurement noise is examined in the present work.

II. Background and Preliminaries

A. Notations, Definitions and Lemmas

The notation $\|\cdot\|$ represents the induced norm of a matrix or the Euclidean norm of a vector. I_n denotes the $n\times n$ identity matrix. For $y_i \in R^{m_i}, i=1,\cdots,n$, $\mathrm{col}(y_1,\cdots,y_n) = [y_1^T,\cdots,y_n^T]^T$. Given $\alpha > 0$ and $x \in R^n$, denote $\mathrm{sig}^\alpha(x) = \mathrm{col}(\mathrm{sig}^\alpha(x_1),\cdots,\mathrm{sig}^\alpha(x_n))$, where $\mathrm{sig}^\alpha(x_i) = \mathrm{sgn}(x_i)|x_i|^\alpha (i=1,\cdots,n)$, and $\mathrm{sgn}(\cdot)$ is the signum function. For $x\in R^3$, $x^\times \in R^{3\times 3}$ refers to the skew-symmetric matrix defined by $x^\times = [0,\ -x_3,\ x_2;\ x_3,\ 0,\ -x_1;\ -x_2,\ x_1,\ 0]$.

For any $\lambda > 0$ and any set of real parameters $r_i > 0\ (i=1,\cdots,n)$, a dilation operator $\delta_\lambda^r : R^n \longmapsto R^n$ is defined by $\delta_\lambda^r(x_1,\cdots,x_n) = \mathrm{col}(\lambda^{r_1}x_1,\cdots,\lambda^{r_n}x_n)$, where $r=\mathrm{col}(r_1,\cdots,r_n)$.

A continuous function $V: R^n \longmapsto R$ is homogeneous of degree k with respect to (w.r.t.) the dilation δ_λ^r if $V(\delta_\lambda^r(x)) = \lambda^k V(x),\ \forall\lambda>0$. A differential system $\dot{x}=f(x)$ (or a vector field f), with continuous $f: R^n \longmapsto R^n$, is homogeneous of degree k w.r.t. the dilation δ_λ^r if $f_i(\delta_\lambda^r(x)) = \lambda^{k+r_i} f_i(x),\ i=1,\cdots,n,\ \forall\lambda>0$.

Lemma 1 [32]. Consider the map $\phi: (0,\infty)\times S^n \mapsto R^n\setminus\{0\}$ defined by $\phi(\lambda,x) = \delta_\lambda^r(x)$, where $x\in S^n = \{x \in R^n | \|x\|=1\}$. Then, ϕ is a bijection. Furthermore, denoting its inverse $\phi^{-1}: R^n\setminus\{0\} \mapsto (0,\infty)\times S^n$ by $\psi(y)=\phi^{-1}(y) = (\psi_\lambda(y),\psi_x(y))$, we have that ψ_λ and ψ_x are C^∞ on $R^n\setminus\{0\}$, $\lim_{\|y\|\to 0}\psi_\lambda(y)\to 0$, and $\lim_{\|y\|\to\infty}\psi_\lambda(y)\to\infty$.

Definition 1. Consider the following system:

$$\dot{x} = f(x,t),\ f(0,t)=0,\ x\in\Psi\subseteq R^n \tag{1}$$

where $f: \Psi\times R^+ \mapsto R^n$ is continuous on an open neighborhood Ψ of the origin $x=0$. The origin of system (1) is said to be (locally) fixed-time stable if it is Lyapunov stable and fixed-time convergent in a neighborhood $\Psi_0\subseteq\Psi$ of the origin. The "fixed-time convergence" refers to that for any initial condition $x(t_0)=x_0\in\Psi_0$ at any given initial time t_0, there is a settling time $T>0$ which is independent of initial conditions, such that every solution $x(t;t_0,x_0)$ of system (1) is defined for $t\in[t_0,t_0+T)$, $x(t;t_0,x_0)\in\Psi_0\setminus\{0\}$, for $t>t_0+T$, $x(t;t_0,x_0)=0$ and $\lim_{t\to t_0+T} x(t;t_0,x_0)=0$. If $\Psi=\Psi_0$ is any subset (arbitrarily large) of R^n, then the origin of system (1) is semi-globally fixed-time stable. If $\Psi=\Psi_0=R^n$, then the origin of system (1) is globally fixed-time stable.

Lemma 2 [33]. For any $x_i\in R, i=1,2,\cdots,n$, and a real number $\nu\in(0,1]$, $\left(\sum_{i=1}^n |x_i|\right)^\nu \le \sum_{i=1}^n |x_i|^\nu \le n^{1-\nu}\left(\sum_{i=1}^n |x_i|\right)^\nu$.

Lemma 3 [33]. For any $x_i\in R, i=1,2,\cdots,n$, and a real number $p>1$, $\sum_{i=1}^n |x_i|^p \le \left(\sum_{i=1}^n |x_i|\right)^p \le n^{p-1}\sum_{i=1}^n |x_i|^p$.

Lemma 4 [34]. For any $x\in R, y\in R, c>0$, $d>0$, and $\gamma>0$, $|x|^c|y|^d \le c\gamma|x|^{c+d}/(c+d) + d|y|^{c+d}/(\gamma^{c/d}(c+d))$.

Lemma 5. Consider system (1). Suppose that there exists a positive definite continuous function $V(x)$ defined on Ψ and it satisfies $\dot{V}(x) \le \begin{cases} -k_1V^\beta & \text{if } V>1 \\ -k_2V^\alpha & \text{if } V\le 1\end{cases}$, where $k_1>0, k_2>0, \beta>1$ and $0<\alpha<1$. Then the origin of system (1) is fixed-time stable. The settling time $T(x_0)$ satisfies $T(x_0)\le 1/[k_1(\beta-1)] + 1/[k_2(1-\alpha)], \forall x_0\in\Psi$.

Proof. See [22].

Remark 1. Definition 1 is a modification of the definition of "finite-time stable" given in [35] since fixed-time stability can be considered as a special case of finite-time stability. Furthermore, semi-global stabilization implies that any given subset of R^n (no matter how large it is) can be included in the region of attraction, but this is not true for local stabilization.

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B. Spacecraft Attitude Kinematics and Dynamics

The equations of motion for rigid spacecraft are [36]

$$\dot{q} = P(q)\omega \tag{2}$$
$$J\dot{\omega} = -\omega^{\times} J\omega + \tau \tag{3}$$

where $\omega \in R^3$ is the angular velocity of the spacecraft with respect to an inertial frame *I* and expressed in the body frame *B*, $J \in R^{3\times 3}$ is the positive-definite mass moment of inertia matrix, $\tau \in R^3$ is the applied control torque generated by actuators, $q(t) \in R^3$ represents the MRPs [37] describing the spacecraft attitude with respect to an inertial frame, defined by $q(t) = \varrho(t)\tan\left(\frac{\kappa(t)}{4}\right)$, $\kappa \in [0, 2\pi)$rad with ϱ and κ denoting the Euler eigenaxis and eigenangle, respectively. The Jacobian matrix $P(q) \in R^{3\times 3}$ for the MRPs is defined by $P(q) = \frac{1}{2}\left[\frac{1-q^T q}{2} I_3 + q^{\times} + qq^T\right]$ [37].

Let q_d be the desired attitude which is generated by $\dot{q}_d = P(q_d)\omega_d$, where ω_d denotes the desired angular velocity. In this paper, it is assumed that the desired trajectory q_d is constructed such that the singularity problem associated with MRPs is avoided, and that ω_d and $\dot{\omega}_d$ are uniformly bounded. The relative attitude between the actual attitude q and the desired attitude q_d is computed as $q_e = q \otimes q_d^{-1} = \frac{q_d(q^T q - 1) + q(1 - q_d^T q_d) - 2q_d^{\times} q}{1 + q_d^T q_d q^T q + 2q_d^T q}$ [37]. The relative angular velocity is given as $\omega_e = \omega - C_{q_e}\omega_d$, where $C_{q_e} = C(q_e) = I_3 + \frac{8(q_e^{\times})^2 - 4(1 - q_e^T q_e)q_e^{\times}}{(1 + q_e^T q_e)^2}$ denotes the corresponding direction cosine matrix relate to q_e. Then, the dynamic equations for the attitude tracking error q_e and angular velocity error ω_e are:

$$\dot{q}_e = P(q_e)\omega_e \tag{4}$$
$$\begin{aligned} J\dot{\omega}_e &= -\omega^{\times} J\omega + \tau - JC_{q_e}\dot{\omega}_d + J\omega_e^{\times} C_{q_e}\omega_d \\ &= \tau + \bar{f}(\omega_e, \omega_d, \dot{\omega}_d) \end{aligned} \tag{5}$$

where $\bar{f}(\omega_e, \omega_d, \dot{\omega}_d) = -(\omega_e + C_{q_e}\omega_d)^{\times} J(\omega_e + C_{q_e}\omega_d) - JC_{q_e}\dot{\omega}_d + J\omega_e^{\times} C_{q_e}\omega_d$.

By appropriate procedures, the attitude tracking error system given in (4) and (5) can be transformed into

$$\dot{q}_e = v_e,\ \dot{v}_e = \bar{\tau} + f(q_e, v_e, \omega_d, \dot{\omega}_d) \tag{6}$$

with $\bar{\tau} = g(q_e)\tau, g(q_e) = P(q_e)J^{-1}$, and $f(q_e, v_e, \omega_d, \dot{\omega}_d) = -g(q_e)(P^{-1}(q_e)v_e + C_{q_e}\omega_d)^{\times} J(P^{-1}(q_e)v_e + C_{q_e}\omega_d) - P(q_e)C_{q_e}\dot{\omega}_d + P(q_e)(P^{-1}(q_e)v_e)^{\times} C_{q_e}\omega_d + \dot{P}(q_e)P^{-1}(q_e)v_e$.

The main objective of this paper is to develop a velocity-free attitude control law for τ so that the attitude state tracking errors q_e and ω_e converge to zero within fixed time. Note that although the problem of attitude control for rigid spacecraft is addressed in this work, the control law derived here can be directly applied to a more general class of second-order nonlinear systems expressed in the form of (6).

Remark 2. Since the Jacobian function $\partial f/\partial v_e$ is not globally bounded, we can conclude that the nonlinear function $f(q_e, v_e, \omega_d, \dot{\omega}_d)$ does not have a global Lipschitz property, i.e. the globally Lipschitz assumption in [31] does not hold for the spacecraft system studied in this paper. Therefore, we will design a semi-global fixed-time observer rather than a global fixed-time observer. Semi-global implies that there exists a suitable observer gain depending on a compact set (which can be chosen arbitrary large) such that the fixed-time convergence of the observer can be achieved for any initial conditions within this compact set.

Remark 3. In contrast to the usual Rodrigues parameters (i.e. $q(t) = \varrho(t)\tan(\kappa(t)/2)$ [37]), the advantage of the MRPs-based attitude description is that it is valid for eigenaxis rotations up to $360°$. Although the unit quaternion can globally represent the attitude of a spacecraft without singularities, a norm constraint is imposed on the four parameters. Thus, the MRPs are employed to represent the spacecraft attitude in this paper.

III. Velocity-Free Fixed-Time Attitude Controller Design

To design a velocity-free fixed-time attitude controller, a semi-global observer is proposed in this section so that the observer errors can converge to zero within fixed time when there are no disturbances and parametric uncertainties. The effect of uncertainties is also discussed. Then, a velocity-free fixed-time attitude tracking controller is designed. Finally, the fixed-time convergence of the resulting closed-loop system is analyzed by using the homogeneous Lyapunov approach together with the homogeneity property.

A. Semi-Global Fixed-Time Observer

Let $\hat{q}_e$ and $\hat{v}_e$ be estimates of q_e and v_e, respectively. The fixed-time observer is proposed as follows:

$$\begin{cases} \dot{\hat{q}}_e = \hat{v}_e + \theta\gamma_1\left(\mathrm{sig}^{\alpha_1}(\tilde{q}_e) + \mathrm{sig}^{\beta_1}(\tilde{q}_e)\right) \\ \dot{\hat{v}}_e = \bar{\tau} + \theta^2\gamma_2\left(\mathrm{sig}^{\alpha}(\tilde{q}_e) + \mathrm{sig}^{\beta_2}(\tilde{q}_e)\right) + \hat{f} \end{cases} \tag{7}$$

where $\tilde{q}_e = q_e - \hat{q}_e$, $0 < \alpha < 1$, $\alpha_1 = (1+\alpha)/2$, $\beta_1 = 2 - \alpha_1$, $\beta_2 = 2 - \alpha$, $\hat{f} = f(q_e, \hat{v}_e, \omega_d, \dot{\omega}_d)$, θ and $\gamma_i (i = 1, 2)$ are some positive constants.

The dynamic equations for the observer errors $\tilde{q}_e$ and $\tilde{v}_e = v_e - \hat{v}_e$ are given as follows:

$$\begin{cases} \dot{\tilde{q}}_e = \tilde{v}_e - \theta\gamma_1\left(\mathrm{sig}^{\alpha_1}(\tilde{q}_e) + \mathrm{sig}^{\beta_1}(\tilde{q}_e)\right) \\ \dot{\tilde{v}}_e = -\theta^2\gamma_2\left(\mathrm{sig}^{\alpha}(\tilde{q}_e) + \mathrm{sig}^{\beta_2}(\tilde{q}_e)\right) + (f - \hat{f}). \end{cases} \tag{8}$$

Defining $\eta_1 = \tilde{q}_e$, $\eta_2 = \tilde{v}_e/\theta$, and $\eta = \mathrm{col}(\eta_1, \eta_2)$, we have

$$\dot{\eta} = h_1 + h_2 + h_3 \tag{9}$$

where $h_1(\eta) = \theta\mathrm{col}(\eta_2/2 - \gamma_1\mathrm{sig}^{\alpha_1}(\eta_1), -\gamma_2\mathrm{sig}^{\alpha}(\eta_1))$, $h_2(\eta) = \theta\mathrm{col}(\eta_2/2 - \gamma_1\mathrm{sig}^{\beta_1}(\eta_1), -\gamma_2\mathrm{sig}^{\beta_2}(\eta_1))$, and $h_3 = \mathrm{col}(0, (f - \hat{f})/\theta)$. Consider the dilations $\delta_{1\lambda}^r(\eta) = \mathrm{col}(\lambda\eta_1, \lambda^{\alpha_1}\eta_2)$ and $\delta_{2\lambda}^r(\eta) = \mathrm{col}(\lambda\eta_1, \lambda^{\beta_1}\eta_2)$. Then, we can demonstrate that $h_1(\eta)$ is homogeneous of degree $(\alpha-1)/2 < 0$ w.r.t. the dilation $\delta_{1\lambda}^r(\eta)$ and $h_2(\eta)$ is homogeneous of degree $(1-\alpha)/2 > 0$ w.r.t. the dilation $\delta_{2\lambda}^r(\eta)$.

Denote $M_1 \in R^{6\times 6}$ as $M_1 = [-\gamma_1 I_3,\ I_3/2;\ -\gamma_2 I_3,\ 0]$. Since M_1 is a Hurwitz matrix, there exists a positive definite matrix $N_1 = N_1^T$ such that $M_1^T N_1 + N_1 M_1 = -I_6$. Define $x_1 = \eta_1, x_2 = \mathrm{sig}^{1/\alpha_1}(\eta_2)$, $x = \mathrm{col}(x_1, x_2)$, $V_x = \|x\|$, $y_1 = \eta_1, y_2 = \mathrm{sig}^{1/\beta_1}(\eta_2)$, $y = \mathrm{col}(y_1, y_2)$, and $V_y = \|y\|$. We can obtain that V_x and V_y are homogeneous of degree 1 w.r.t. the dilation $\delta_{1\lambda}^r(\eta)$ and the dilation $\delta_{2\lambda}^r(\eta)$, respectively.

Denote $V(\eta) = \eta^T N_1 \eta$. Like the works in [32], [38], the candidate Lyapunov function used in this paper is given in the following propositions.

Proposition 1: Let $\phi(z) \in C^\infty(R, R)$ be

$$\phi(z) = \begin{cases} 0 & \text{if } z \in (-\infty, 1] \\ 1 & \text{if } z \in [2, +\infty) \end{cases} \quad \text{and} \quad \frac{d\phi}{dz} \geq 0,\ \forall z \in R. \quad (10)$$

Consider

$$\bar{V}_1(\eta) = \int_{0^+}^{+\infty} \frac{1}{\rho^3} \phi(V(\delta_{1\rho}^r(\eta))) d\rho. \quad (11)$$

If $\eta \in R^6 \setminus \{0\}$ and $\bar{V}_1(0) = 0$, then there exists an $\epsilon > 0$ such that for all $\alpha \in (1-\epsilon, 1+\epsilon)$ the function $\bar{V}_1(\eta)$ is well defined, positive definite, radially unbounded, of class $C^1(R^6, R)$, and satisfies

(a) $\bar{V}_1(\eta)$ is homogeneous of degree 2 w.r.t. the dilation $\delta_{1\lambda}^r(\eta)$;

(b) there exist some constants $c_1 > 0$ and $c_2 > 0$ such that $c_1 V_x^2 \leq \bar{V}_1(\eta) \leq c_2 V_x^2$, for all $\eta \in R^6$, where $c_1 = \min_{\eta \in S^6(x)} \left(\bar{V}_1(\eta)\right)$ and $c_2 = \max_{\eta \in S^6(x)} \left(\bar{V}_1(\eta)\right)$ with $S^6(x) = \{\eta \in R^6 | \|x(\eta)\| = 1\}$;

(c) $L_{h_1} \bar{V}_1(\eta)$ is homogeneous of degree $(\alpha+3)/2$ w.r.t. the dilation $\delta_{1\lambda}^r(\eta)$ and satisfies $L_{h_1} \bar{V}_1(\eta) \leq -\varphi\theta \bar{V}_1^{(\alpha+3)/4}(\eta)$, for all $\eta \in R^6$, where $\varphi > 0$;

(d) $\frac{\partial \bar{V}_1(\eta)}{\partial \eta_{1i}}$ is homogeneous of degree 1 and $\frac{\partial \bar{V}_1(\eta)}{\partial \eta_{2i}}$ is homogeneous of degree $2 - \alpha_1$ w.r.t. the dilation $\delta_{1\lambda}^r(\eta)$, respectively, where $i = 1, 2, 3$. Furthermore, there exist some constants $c_{3i} > 0$ and $c_{4i} > 0$ such that $\left| \frac{\partial \bar{V}_1(\eta)}{\partial \eta_{1i}} \right| \leq c_{3i} V_x$ and $\left| \frac{\partial \bar{V}_1(\eta)}{\partial \eta_{2i}} \right| \leq c_{4i} V_x^{2-\alpha_1}$, where $c_{3i} = \max_{\eta \in S^6(x)} \left(\left| \frac{\partial \bar{V}_1(\eta)}{\partial \eta_{1i}} \right| \right)$ and $c_{4i} = \max_{\eta \in S^6(x)} \left(\left| \frac{\partial \bar{V}_1(\eta)}{\partial \eta_{2i}} \right| \right)$.

Proof. See the Appendix.

A similar proposition is presented as follows.

Proposition 2: Denote

$$\bar{V}_2(\eta) = \int_{0^+}^{+\infty} \frac{1}{\rho^3} \phi(V(\delta_{2\rho}^r(\eta))) d\rho. \quad (12)$$

If $\eta \in R^6 \setminus \{0\}$ and $\bar{V}_2(0) = 0$, then there exists an $\epsilon > 0$ such that for all $\alpha \in (1-\epsilon, 1+\epsilon)$ the function $\bar{V}_2(\eta)$ is well defined, positive definite, radially unbounded, of class $C^1(R^6, R)$, and satisfies

(a) $\bar{V}_2(\eta)$ is homogeneous of degree 2 w.r.t. the dilation $\delta_{2\lambda}^r(\eta)$;

(b) there exist some constants $c_5 > 0$ and $c_6 > 0$ such that $c_5 V_y^2 \leq \bar{V}_2(\eta) \leq c_6 V_y^2$, for all $\eta \in R^6$.

(c) $L_{h_2} \bar{V}_2(\eta)$ is homogeneous of degree $(5-\alpha)/2$ w.r.t. the dilation $\delta_{2\lambda}^r(\eta)$ and satisfies $L_{h_2} \bar{V}_2(\eta) \leq -\varphi\theta \bar{V}_2^{(5-\alpha)/4}(\eta)$, for all $\eta \in R^6$, where $\varphi > 0$;

(d) $\frac{\partial \bar{V}_2(\eta)}{\partial \eta_{1i}}$ is homogeneous of degree 1 and $\frac{\partial \bar{V}_2(\eta)}{\partial \eta_{2i}}$ is homogeneous of degree $2 - \beta_1$ w.r.t. the dilation $\delta_{2\lambda}^r(\eta)$, respectively, where $i = 1, 2, 3$. Furthermore, there exist some constants $c_{7i} > 0$ and $c_{8i} > 0$ such that $\left| \frac{\partial \bar{V}_2(\eta)}{\partial \eta_{1i}} \right| \leq c_{7i} V_y$ and $\left| \frac{\partial \bar{V}_2(\eta)}{\partial \eta_{2i}} \right| \leq c_{8i} V_y^{2-\beta_1}$.

Remark 4. Equation (10) gives the condition for choosing $\phi(\cdot)$, and any function satisfying (10) can be used for $\phi(\cdot)$. An example for $\phi(\cdot)$ is $\phi(z) = \frac{s(z-1)}{s(z-1)+s(2-z)}$, where $s(z)$ is defined as $s(z) = 0$ for $z \in (-\infty, 0]$ and $s(z) = e^{-1/z}$ for $z \in (0, +\infty)$. Since $s(z)$ is C^∞, we can conclude that $\phi(z) \in C^\infty$. Furthermore, we can verify that the function $\phi(z)$ meets the condition presented in (10).

The fixed-time convergence of the observer in (7) is now given in the following theorem.

Theorem 1. Consider the observer (7) with bounded control torque τ. For any $\Delta_1 > 0$, if $E = \text{col}(q_e, \omega_e, \hat{q}_e, \hat{\omega}_e)$ lies within the compact set $\Omega_{\Delta_1} = \{E \in R^{12} | \|E\| \leq \Delta_1\}$, then there exist an observer parameter θ and an $\epsilon > 0$ such that the observer errors $\tilde{q}_e$ and $\tilde{v}_e$ converge to zero within fixed time, for all $\alpha \in (1-\epsilon, 1)$.

Proof. The proof is divided into two parts: Part 1 ($V_y > 1$) and Part 2 ($V_y \leq 1$).

Part 1: $V_y > 1$. The candidate Lyapunov function is $V_2(\eta) = \bar{V}_2(\eta)/c_5$, where $\bar{V}_2(\eta)$ is defined in (12) and $c_5 > 0$ is given in Proposition 2. Clearly, $V_2(\eta) > 1$ when $V_y > 1$. Using (9), the time derivative of $\bar{V}_2(\eta)$ is

$$\dot{\bar{V}}_2(\eta) = L_{h_1} \bar{V}_2(\eta) + L_{h_2} \bar{V}_2(\eta) + \frac{\partial \bar{V}_2}{\partial \eta} h_3. \quad (13)$$

Note that $L_{h_1} \bar{V}_2(\eta)$ is continuous and when $\alpha = 1$, we have $\delta_{2\rho}^r(\eta) = \rho\eta$ and $V(\delta_{2\rho}^r(\eta)) = \rho^2 V(\eta) = \rho^2 \eta^T N_1 \eta$, and consequently $L_{h_1} V(\eta) = \frac{\partial V(\eta)}{\partial \eta} h_1 = -\theta \eta^T \eta$. Thus, we can obtain

$$\begin{aligned} L_{h_1} \bar{V}_2(\eta) &= \int_{0^+}^{\infty} \frac{1}{\rho^3} \phi'(\rho^2 V(\eta))(-\rho^2 \theta \eta^T \eta) d\rho \\ &= -\theta \eta^T \eta \int_{0^+}^{\infty} \frac{1}{\rho} \phi'(\rho^2 V(\eta)) d\rho < 0 \end{aligned}$$

which in turn implies that there exists an $\epsilon_1 \in (0, 1)$ such that $L_{h_1} \bar{V}_2(\eta) \leq 0$ for all $\alpha \in (1-\epsilon_1, 1)$ and all $\eta \in \{\eta \in R^6 | V_{y(\eta)} > 1\}$. Furthermore, by use of Proposition 2, we can obtain that there exist $\epsilon_2 \in (0, 1)$ and $\varphi_1 > 0$ such that $L_{h_2} \bar{V}_2(\eta) \leq -\varphi_1 \theta \bar{V}_2^{(5-\alpha)/4}$ for all $\alpha \in (1-\epsilon_2, 1)$ and all $\eta \in R^6$. Therefore, there exists $\epsilon_3 = \min(\epsilon_1, \epsilon_2)$ such that for all $\alpha \in (1-\epsilon_3, 1)$

$$\dot{\bar{V}}_2(\eta) \leq -\varphi_1 \theta \bar{V}_2^{(5-\alpha)/4} + \frac{\partial \bar{V}_2}{\partial \eta} h_3. \quad (14)$$

If $E \in \Omega_{\Delta_1}$, by the mean value theorem, then we can obtain that there exists a constant $\psi > 0$ so that $\|(f - \hat{f})/\theta\| \leq \psi \|\tilde{v}_e\|/\theta = \psi \|\eta_2\| \leq \psi V_y^{\beta_1}$, where Lemma 3 has been applied. Then, (14) becomes

$$\begin{aligned} \dot{\bar{V}}_2(\eta) &\leq -\varphi_1 \theta \bar{V}_2^{(5-\alpha)/4} + \sum_{i=1}^{3} c_{8i} \psi V_y^{2-\beta_1} \|\eta_2\| \\ &\leq -\varphi_1 \theta \bar{V}_2^{(5-\alpha)/4} + c_8 \psi V_y^2 \\ &\leq -\varphi_1 \theta \bar{V}_2^{(5-\alpha)/4} + \frac{c_8 \psi}{c_5} \bar{V}_2^{(5-\alpha)/4} \\ &= -\varphi_1 \left(\theta - \frac{c_8 \psi}{\varphi_1 c_5} \right) \bar{V}_2^{(5-\alpha)/4} \end{aligned} \quad (15)$$

where $c_8 = \sum_{i=1}^{3} c_{8i}$, and Proposition 2 has been used. Thus, the time derivative of $V_2(\eta)$ is

$$\begin{aligned} \dot{V}_2(\eta) &\leq -\varphi_1 c_5^{(1-\alpha)/4} \left(\theta - \frac{c_8 \psi}{\varphi_1 c_5} \right) V_2^{(5-\alpha)/4} \\ &= -\varrho_1 V_2^{(5-\alpha)/4}. \end{aligned} \quad (16)$$

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If we select the parameter θ so that $\theta > c_8\psi/(\varphi_1 c_5)$ (i.e. $\varrho_1 > 0$), then it follows from (16) and the proof of Lemma 5 that $V_2(\eta)$ will converge to $V_2(\eta) \le 1$ (i.e. $V_y \le 1$) within fixed time $t_1 = 4/[\varrho_1(1-\alpha)]$.

Part 2: $V_y \le 1$. When $V_y \le 1$, it can be verified that $V_x \le 1$. Consider the Lyapunov function $V_1(\eta) = \bar{V}_1(\eta)/c_2$, where $\bar{V}_1(\eta)$ and c_2 are defined in Proposition 1, which implies that $V_1(\eta) \le 1$ when $V_y \le 1$. Following the same procedure in Part 1 and using Proposition 1, there exist $\epsilon_4 \in (0,1)$ and $\varphi_2 > 0$ such that

$$\begin{aligned}\dot{\bar{V}}_1(\eta) &\le -\varphi_2\theta\bar{V}_1^{(3+\alpha)/4} + \frac{\partial \bar{V}_1}{\partial \eta}h_3 \\ &\le -\varphi_2\theta\bar{V}_1^{(\alpha+3)/4} + \sum_{i=1}^{3} c_{4i}\psi V_x^{2-\alpha_1}\|\eta_2\| \\ &\le -\varphi_2\theta\bar{V}_1^{(\alpha+3)/4} + 3^{(1-\alpha_1)/2}c_4\psi V_x^2 \\ &\le -\varphi_2\theta\bar{V}_1^{(\alpha+3)/4} + \frac{3^{(1-\alpha_1)/2}c_4\psi}{c_1}\bar{V}_1 \end{aligned} \tag{17}$$

where $c_4 = \sum_{i=1}^{3} c_{4i}$ and we have used the fact that $\|\eta_2\| \le 3^{(1-\alpha_1)/2}V_x^{\alpha_1}$ obtained by Lemma 2. Hence, the time derivative of $V_1(\eta)$ is

$$\begin{aligned}\dot{V}_1(\eta) &\le -\varphi_2\theta c_2^{(\alpha-1)/4}V_1^{(\alpha+3)/4} + \frac{3^{(1-\alpha_1)/2}c_4\psi}{c_1}V_1 \\ &\le -\varphi_2 c_2^{(\alpha-1)/4}\left[\theta - \frac{3^{(1-\alpha_1)/2}c_4\psi}{c_1\varphi_2 c_2^{(\alpha-1)/4}}\right]V_1^{(\alpha+3)/4} \\ &= -\varrho_2 V_1^{(\alpha+3)/4}. \end{aligned} \tag{18}$$

If we choose the parameter θ such that $\theta > 3^{(1-\alpha_1)/2}c_4\psi/(c_1\varphi_2 c_2^{(\alpha-1)/4})$ (i.e. $\varrho_2 > 0$), then we can conclude from (18) that $V_1(\eta)$ will converge to zero when $t \to t_1 + 4/[\varrho_2(1-\alpha)]$.

With the combination of Part 1 and Part 2, it can be obtained that the observer errors $\tilde{q}_e$ and $\tilde{\omega}_e$ will converge to the origin within fixed time $t = 4/[\varrho_2(1-\alpha)] + 4/[\varrho_1(1-\alpha)]$ for all $\alpha \in (1-\epsilon, 1)$ with $\epsilon = \min(\epsilon_3, \epsilon_4)$. This completes the proof.

Next, the effect of both parametric and non-parametric uncertainties on the performance of the fixed-time observer is addressed. In this case, the spacecraft dynamics in (3) is reexpressed as $J\dot{\omega} = -\omega^{\times}J\omega + \tau + \vartheta$, where ϑ is the bounded external disturbance, and the inertia matrix is assumed to be $J = J_0 + \Delta J$ with J_0 and ΔJ representing respectively the nominal part and the uncertain part of the inertia matrix. By replacing J with J_0, the fixed-time observer in (7) becomes

$$\begin{cases} \dot{\hat{q}}_e = \hat{v}_e + \theta\gamma_1\left(\mathrm{sig}^{\alpha_1}(\tilde{q}_e) + \mathrm{sig}^{\beta_1}(\tilde{q}_e)\right) \\ \dot{\hat{v}}_e = g_0\tau + \theta^2\gamma_2\left(\mathrm{sig}^{\alpha}(\tilde{q}_e) + \mathrm{sig}^{\beta_2}(\tilde{q}_e)\right) + \hat{f}_0 \end{cases} \tag{19}$$

and the dynamics of the observer errors is

$$\begin{cases} \dot{\tilde{q}}_e = \tilde{v}_e - \theta\gamma_1\left(\mathrm{sig}^{\alpha_1}(\tilde{q}_e) + \mathrm{sig}^{\beta_1}(\tilde{q}_e)\right) \\ \dot{\tilde{v}}_e = -\theta^2\gamma_2\left(\mathrm{sig}^{\alpha}(\tilde{q}_e) + \mathrm{sig}^{\beta_2}(\tilde{q}_e)\right) + f_0 - \hat{f}_0 + \Upsilon \end{cases} \tag{20}$$

where $\Upsilon = g\tau - g_0\tau + \vartheta + f - f_0$ denotes the lumped uncertainty and the definition of f_0 is similar to that of f in which J is replaced with J_0.

Corollary 1. Consider system (6) in the presence of parametric uncertainties and bounded external disturbances and the observer (19) with bounded control torque τ. For any $\Delta_1 > 0$, if E lies within the compact set Ω_{Δ_1}, then there exist an observer parameter $\theta > 1$ and an $\epsilon > 0$ such that for all $\alpha \in (1-\epsilon, 1)$ the observer error $o_e = \mathrm{col}(\tilde{q}_e, \tilde{v}_e)$ converges to the region $\|o_e\| \le \Delta_e = 6^{1-\frac{\alpha_1}{2}}\left(\frac{c_1}{c_2}\right)^{\frac{\alpha_1}{2}}\left(\frac{\Upsilon_M}{\varrho_2 c_2 \theta^{\frac{1-\alpha}{2\alpha_1}}}\right)^{\frac{\alpha_1}{\alpha}}$ within fixed time, where Υ_M is some positive constant.

Proof. See the Appendix.

Remark 5. If the parameter $\theta > 1$ is chosen such that $\Upsilon_M/(\varrho_2 c_2\theta^{(1-\alpha)/(2\alpha_1)}) < 1$ and the parameter α is selected to approximate to zero such that the power $\alpha_1/\alpha = 1/2 + 1/(2\alpha)$ is sufficiently larger than 1, then we obtain that the observer error o_e could be as small as desirable, which implies that the smaller the observer errors, the larger the observer parameter θ and the smaller the observer parameter α are required. Thus, to achieve better disturbance rejection and robustness properties, we can select smaller α rather than larger observer gains. The high-gain observer [39] is robust to both parametric uncertainties and external disturbances with sufficiently large observer gains. However, the high-gain observer suffers from a peaking phenomenon [39].

B. Velocity-Free Fixed-Time Attitude Controller

The velocity-free fixed-time attitude control law is now designed as follows:

$$\begin{aligned}\tau = {} & g^{-1}\left(-\hat{f} - k_1\mathrm{sig}^{\alpha}(q_e) - k_2\mathrm{sig}^{\alpha/\alpha_1}(\hat{v}_e)\right) \\ & - g^{-1}\left(k_1\mathrm{sig}^{\beta_2}(q_e) + k_2\mathrm{sig}^{\beta_2/\beta_1}(\hat{v}_e)\right) \end{aligned} \tag{21}$$

where k_1 and k_2 are some positive constants. Define $\zeta_1 = q_e$, $\zeta_2 = \hat{v}_e$ and $\zeta = \mathrm{col}(\zeta_1, \zeta_2)$. With the control law (21), ζ is governed by the following dynamic equation:

$$\begin{aligned}\dot{\zeta}_1 &= \zeta_2 + \tilde{v}_e \\ \dot{\zeta}_2 &= -k_1\mathrm{sig}^{\alpha}(\zeta_1) - k_2\mathrm{sig}^{\alpha/\alpha_1}(\zeta_2) - k_1\mathrm{sig}^{\beta_2}(\zeta_1) \\ &\quad - k_2\mathrm{sig}^{\beta_2/\beta_1}(\zeta_2) + \theta^2\gamma_2\left(\mathrm{sig}^{\alpha}(\tilde{q}_e) + \mathrm{sig}^{\beta_2}(\tilde{q}_e)\right) \end{aligned} \tag{22}$$

which can be expressed in a compact form as

$$\dot{\zeta} = h_4 + h_5 + h_6 \tag{23}$$

where $h_4 = \mathrm{col}(\zeta_2/2, -k_1\mathrm{sig}^{\alpha}(\zeta_1) - k_2\mathrm{sig}^{\alpha/\alpha_1}(\zeta_2)), h_5 = \mathrm{col}(\zeta_2/2, -k_1\mathrm{sig}^{\beta_2}(\zeta_1) - k_2\mathrm{sig}^{\beta_2/\beta_1}(\zeta_2))$ and $h_6 = \mathrm{col}(\tilde{v}_e, \theta^2\gamma_2(\mathrm{sig}^{\alpha}(\tilde{q}_e) + \mathrm{sig}^{\beta_2}(\tilde{q}_e)))$. Consider the dilations $\delta_{3\lambda}^r(\zeta) = \mathrm{col}(\lambda\zeta_1, \lambda^{\alpha_1}\zeta_2)$ and $\delta_{4\lambda}^r(\zeta) = \mathrm{col}(\lambda\zeta_1, \lambda^{\beta_1}\zeta_2)$. Then, we can verify that h_4 is homogeneous of degree $(\alpha-1)/2 < 0$ w.r.t. the dilation $\delta_{3\lambda}^r(\zeta)$ and h_5 is homogeneous of degree $(1-\alpha)/2 > 0$ w.r.t. the dilation $\delta_{4\lambda}^r(\zeta)$.

Define a Hurwitz matrix $M_2 = [0,\ I_3/2;\ -k_1 I_3,\ -k_2 I_3]$. Hence, there exists a positive definite matrix $N_2 = N_2^T$ such that $M_2^T N_2 + N_2 M_2 = -I_6$. Denote $u_1 = \zeta_1, u_2 = \mathrm{sig}^{1/\alpha_1}(\zeta_2), u = \mathrm{col}(u_1, u_2), v_1 = \zeta_1, v_2 = \mathrm{sig}^{1/\beta_1}(\zeta_2), v = \mathrm{col}(v_1, v_2), U_u = \|u\|$ and $U_v = \|v\|$. Then, it can be obtained that U_u and U_v are homogeneous of degree 1 w.r.t. the dilations $\delta_{3\lambda}^r(\zeta)$ and $\delta_{4\lambda}^r(\zeta)$, respectively.

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Define a radially unbounded function $U = \zeta^T N_2 \zeta$, and construct Lyapunov functions as

$$\bar{U}_1(\zeta) = \int_{0^+}^{+\infty} \frac{1}{\rho^3} \phi(U(\delta^r_{3\rho}(\zeta))) d\rho \tag{24}$$

and

$$\bar{U}_2(\zeta) = \int_{0^+}^{+\infty} \frac{1}{\rho^3} \phi(U(\delta^r_{4\rho}(\zeta))) d\rho. \tag{25}$$

Similar to Propositions 1 and 2, we can show that there exist some constants $d_1 > 0$ and $d_2 > 0$ such that $d_1 U_u^2 \leq \bar{U}_1(\zeta) \leq d_2 U_u^2$ and some constants $d_5 > 0$ and $d_6 > 0$ such that $d_5 U_v^2 \leq \bar{U}_2(\zeta) \leq d_6 U_v^2$. Now, the theorem about the fixed-time observer and the fixed-time velocity-free controller is stated as follows.

Theorem 2. Consider the spacecraft system described by (2) and (3). For any positive constant Δ_2, if the observer is given by (7), the control law is defined by (21), and the initial conditions $\eta(0)$ and $\zeta(0)$ satisfy

$$\max(1, V_2(\eta)) + \max(1, U_2(\zeta)) \leq \Delta_2 \tag{26}$$

where $U_2(\zeta) = \bar{U}_2(\zeta)/d_5$, then there exist an observer parameter θ and an $\epsilon \in (0, 1)$ such that for all $\alpha \in (1 - \epsilon, 1)$

(1) all signals of the resulting closed-loop system are bounded;

(2) the attitude tracking errors (i.e. q_e and ω_e) and the observer errors (i.e. $\tilde{q}_e$ and $\tilde{\omega}_e$) converge to zero within fixed time.

Proof. (1) To show the boundedness of all signals of the closed-loop system, we consider four cases in the proof; that is, Case 1 ($V_y > 1, U_v > 1$), Case 2 ($V_y > 1, U_v \leq 1$), Case 3 ($V_y \leq 1, U_v > 1$), and Case 4 ($V_y \leq 1, U_v \leq 1$). Note that all signals of the closed-loop system are bounded for Case 4. Thus, only Cases 1-3 are required to address.

Case 1: $V_y > 1$ and $U_v > 1$. Consider the Lyapunov function candidate $L_1 = U_2(\zeta) + KV_2(\eta)$. By the proof of Theorem 1, we know that there exists $\epsilon_1 \in (0, 1)$ such that for all $\alpha \in (1 - \epsilon_1, 1)$

$$\dot{V}_2(\eta) \leq -\varrho_1 V_2^{(5-\alpha)/4}. \tag{27}$$

Following the same procedure of the proof of Theorem 1, we can obtain that there exist a positive constant ϱ_3 and $\epsilon_2 \in (0, 1)$ such that for all $\alpha \in (1 - \epsilon_2, 1)$

$$\dot{\bar{U}}_2(\zeta) \leq -\bar{\varrho}_3 U_2^{(5-\alpha)/4} + \frac{\partial \bar{U}_2}{\partial \zeta} h_6 \tag{28}$$

where $\bar{\varrho}_3 = \varrho_3 d_5^{(5-\alpha)/4}$. Note that $\frac{\partial \bar{U}_2(\zeta)}{\partial \zeta_{1i}}$ is homogeneous of degree 1 and $\frac{\partial \bar{U}_2(\zeta)}{\partial \zeta_{2i}}$ is homogeneous of degree $2 - \beta_1$ w.r.t. the dilation $\delta^r_{4\lambda}(\zeta)$, respectively, where $i = 1, 2, 3$. Further, there exist positive constants d_{7i} and d_{8i} such that $\left|\frac{\partial \bar{U}_2(\zeta)}{\partial \zeta_{1i}}\right| \leq d_{7i} U_v$ and $\left|\frac{\partial \bar{U}_2(\zeta)}{\partial \zeta_{2i}}\right| \leq d_{8i} U_v^{2-\beta_1}$. Using the inequalities $\|\text{sig}^\alpha(\tilde{q}_e)\| \leq 3^{(1-\alpha)/2} \|\tilde{q}_e\|^\alpha \leq 3^{(1-\alpha)/2} V_2^{\alpha/2}$, $\|\text{sig}^{\beta_2}(\tilde{q}_e)\| \leq \|\tilde{q}_e\|^{\beta_2} \leq V_2^{\beta_2/2}$, and $\|\tilde{v}_e\| = \theta \|\eta_2\| \leq \theta V_2^{\beta_1/2}$, where Lemmas 2 and 3 have been used, it follows that

$$\begin{aligned}
\frac{\partial \bar{U}_2}{\partial \zeta} h_6 &\leq d_8 \gamma_2 \theta^2 U_v^{2-\beta_1} (\|\text{sig}^\alpha(\tilde{q}_e)\| + \|\text{sig}^{\beta_2}(\tilde{q}_e)\|) \\
&\quad + d_7 U_v \|\tilde{v}_e\| \\
&\leq d_8 \gamma_2 \theta^2 U_2^{\frac{2-\beta_1}{2}} \left(3^{\frac{1-\alpha}{2}} V_2^{\frac{\alpha}{2}} + V_2^{\frac{\beta_2}{2}}\right) + d_7 \theta U_2^{\frac{1}{2}} V_2^{\frac{\beta_1}{2}} \\
&\leq \frac{\bar{\varrho}_3}{4} U_2^{\frac{1+3\alpha}{4}} + \varsigma_1 V_2^{\frac{1+3\alpha}{4}} + \frac{\bar{\varrho}_3}{4} U_2^{\frac{5-\alpha}{4}} + \varsigma_2 V_2^{\frac{5-\alpha}{4}} \\
&\quad + \frac{\bar{\varrho}_3}{4} U_2^{\frac{5-\alpha}{4}} + \varsigma_3 V_2^{\frac{5-\alpha}{4}} \leq \frac{3\bar{\varrho}_3}{4} U_2^{\frac{5-\alpha}{4}} + \varsigma V_2^{\frac{5-\alpha}{4}}
\end{aligned} \tag{29}$$

where Lemma 4 has been used, $d_7 = \sum_{i=1}^3 d_{7i}, d_8 = \sum_{i=1}^3 d_{8i}$, $\varsigma = \varsigma_1 + \varsigma_2 + \varsigma_3$, $\varsigma_1 = 2\alpha/[(1 + 3\alpha)\iota_1^{(2-\beta_1)/\alpha}]$ with $\iota_1 = \bar{\varrho}_3(1+3\alpha)/[4 \times 3^{(1-\alpha)/2} d_8 \gamma_2 \theta^2 (4-2\beta_1)]$, $\varsigma_2 = 2\beta_2/[(5-\alpha)\iota_2^{(2-\beta_1)/\beta_2}]$ with $\iota_2 = \bar{\varrho}_3(5 - \alpha)/[4 d_8 \gamma_2 \theta^2 (4 - 2\beta_1)]$, and $\varsigma_3 = 2\beta_1/[(5 - \alpha)\iota_3^{1/\beta}]$ with $\iota_3 = \bar{\varrho}_3(5 - \alpha)/(8 d_7 \theta)$.

Using (27), (28) and (29), the time derivative of L_1 can be obtained as

$$\begin{aligned}
\dot{L}_1 &\leq -\varrho_1 K V_2^{(5-\alpha)/4} - \frac{3\bar{\varrho}_3}{4 d_5} U_2^{\frac{5-\alpha}{4}} + \frac{\varsigma}{d_5} V_2^{\frac{5-\alpha}{4}} \\
&= -\frac{\bar{\varrho}_3}{4 d_5} U_2^{\frac{5-\alpha}{4}} - \varrho_1 \left(K - \frac{\varsigma}{d_5 \varrho_1}\right) V_2^{\frac{5-\alpha}{4}} \leq 0
\end{aligned} \tag{30}$$

if K is chosen such that $K > \varsigma/(d_5 \varrho_1)$, which in turn implies that all signals in the closed-loop system are bounded.

For Case 2 ($V_y > 1, U_v \leq 1$) and Case 3 ($V_y \leq 1, U_v > 1$), we can consider the Lyapunov function $L_2 = U_1(\zeta) + KV_2(\eta)$ and $L_3 = U_2(\zeta) + KV_1(\eta)$, where $U_1(\zeta) = \bar{U}_1(\zeta)/d_2$. Following the procedure of Case 1, the boundedness of all signals of the closed-loop system can be obtained.

(2) As the boundedness of all signals of the system is ensured, by using Theorem 1, we can obtain that the observer errors $\tilde{q}_e$ and $\tilde{\omega}_e$ converge to zero within fixed time t_1. When $t > t_1$, the dynamic equation of ζ in (23) becomes $\dot{\zeta} = h_4 + h_5$. Since h_4 is homogeneous of degree $(\alpha - 1)/2 < 0$ w.r.t. the dilation $\delta^r_{3\lambda}(\zeta)$ and h_5 is homogeneous of degree $(1 - \alpha)/2 > 0$ w.r.t. the dilation $\delta^r_{4\lambda}(\zeta)$, respectively, we can verify that ζ will converge to zero within fixed time t_2. Therefore, $\tilde{q}_e$, $\tilde{\omega}_e$, q_e and $\hat{\omega}_e$ will converge to zero in fixed time $t = t_1 + t_2$, which in turn implies that the attitude tracking errors q_e and ω_e also converge to zero within fixed time. The proof is complete.

Remark 6. In Theorem 2, it is assumed that the initial conditions lie within a bounded set which can be chosen arbitrary large. This implies that there is no singularity for the initial attitude. However, by Theorem 2, we can conclude that the singularity will never occur if the assumption about the initial conditions is satisfied.

Remark 7. If α is selected to be $\alpha = 1$, then the fixed-time observer (7) reduces to a Luenberger-style observer [40] $\begin{cases} \dot{\hat{q}}_e = \hat{v}_e + 2\theta\gamma_1 \tilde{q}_e \\ \dot{\hat{v}}_e = \bar{\tau} + 2\theta^2 \gamma_2 \tilde{q}_e + \hat{f} \end{cases}$, and the control law (21) becomes an asymptotic controller as $\tau = g^{-1}\left(-\hat{f} - 2k_1 q_e - 2k_2 \hat{v}_e\right)$ which can be easily implemented in practice. The only difference between the above method and the proposed approach is the use of an extra parameter α in the observer and

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controller. With appropriate α, the proposed fixed-time control scheme can provide better disturbance rejection and robustness properties without increasing the observer and controller gains.

Remark 8. If the inertia matrix is not exactly known and there are bounded external disturbances, using the fixed-time observer (19), then the fixed-time controller (21) would be

$$\tau = g_0^{-1}\left(-\hat{f}_0 - k_1\text{sig}^{\alpha}(q_e) - k_2\text{sig}^{\alpha/\alpha_1}(\hat{v}_e)\right) - g_0^{-1}\left(k_1\text{sig}^{\beta_2}(q_e) + k_2\text{sig}^{\beta_2/\beta_1}(\hat{v}_e)\right). \quad (31)$$

In this case, the attitude tracking errors will converge to a small bounded region rather than approach to zero within fixed time.

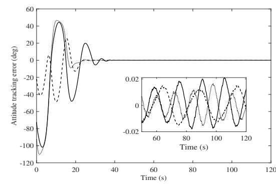
Remark 9. When full-state measurements are available, the terminal sliding mode method may be used to design an attitude controller without requiring the knowledge of inertia matrices (see, e.g. [14], [15]), but the implementation of the proposed control scheme may rely on a known inertia matrix (at least its nominal value). Fortunately, the nominal value of the inertia matrix is usually known in practical situations. In addition, the proposed controller is not applicable to the case in which the initial Euler eigenangle is $\varrho(0) = 2\pi$ (i.e. the initial attitude tracking error in terms of MRPs is not bounded). In this case, we may use a quaternion-based output feedback controller (e.g. [4]) to drive the attitude of spacecraft away from the singular point, and then switch to the proposed controller to force the attitude tracking errors to zero or the neighborhood of zero within fixed time.

Remark 10. If the attitude of a spacecraft is controlled by reaction wheels, the torque is defined by $\tau = -\dot{h}_w - \omega^{\times}h_w$ [41], where $h_w = J_w\Omega_s$ represents the wheel angular momentum, the cross coupling term $\omega^{\times}h_w$ results from gyroscopic effects of the spinning wheels, $J_w = \text{diag}(J_{w1}, J_{w2}, J_{w3})$ is the axial moments of inertia of the wheels, and Ω_s denotes the axial angular velocity of the wheels in regard to the spacecraft. In this case, $\dot{h}_w$ can be considered as the control input, and it can be designed as $\dot{h}_w = -\tau - (P^{-1}\hat{v}_e + C_{q_e}\omega_d)^{\times}h_w$, where τ is defined in (21) or (31).

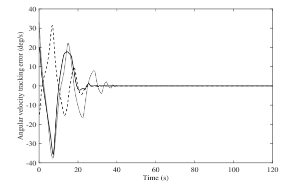
Remark 11. Since the finite-time (fixed-time) convergence of the observer is achieved, the separation principle is satisfied [42]; that is, we can design the observer and the controller separately. The only requirement is that the boundedness of the states of both the observer and the spacecraft system at any time interval $[0, t]$ should be guaranteed. Thus, to verify that the requirement for the separation principle is satisfied, there are two steps in the proof of Theorem 2. In Step 1, it is shown that all signals (i.e., the observer error o_e and the attitude tracking errors q_e and v_e) of the closed-loop system and consequently the control torque τ are bounded for all $t \geq 0$ if the initial conditions lie within a bounded set, i.e. there is no finite time escape. Then, in Step 2, it follows from Theorem 1 that the observer error o_e converges to zero within fixed time t_1, and when $t > t_1$, it can be proven that the attitude tracking errors q_e and v_e converge to zero within fixed time t_2 by using the homogeneity property.

IV. NUMERICAL SIMULATIONS

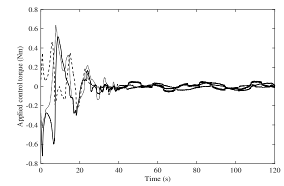
The effectiveness of the proposed controller will be illustrated through numerical simulations in this section. The inertia matrix is considered to be $J = J_0 + \Delta J$, where $J_0 = [1.9\ 0.3\ 0.4; 0.3\ 1.5\ 0.2; 0.4\ 0.2\ 1.3] kg \cdot m^2$ and $\Delta J = 0.1J_0$ denote the nominal part and the uncertain part of the inertia matrix, respectively. The external disturbance ϑ is assumed to be $\vartheta = 10\text{col}(\sin(t/10), \cos(t/10), \sin(t/5))$ mNm. The axial inertia moment matrix of the wheels is $J_w = \text{diag}(0.002, 0.002, 0.002)\text{kg}\cdot\text{m}^2$. Sun sensors are considered to measure the spacecraft attitude. The reference attitude is $q_d = 0.1\text{col}(\cos(0.2t), \sin(0.2t), \sqrt{3})$. The observer and controller parameters are chosen as $\alpha = 0.3, \theta = 2, \gamma_1 =$



(a) Attitude tracking error in Euler angles: φ_e (solid line), θ_e (dashed line), and ψ_e (dotted line)

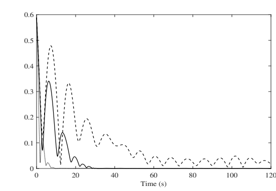


(b) Angular velocity tracking error: ω_{e1} (solid line), ω_{e2} (dashed line), and ω_{e3} (dotted line)

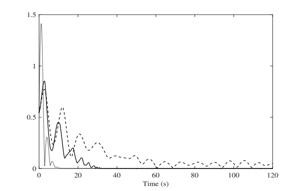


(c) Control torque: τ_1 (solid line), τ_2 (dashed line), and τ_3 (dotted line)

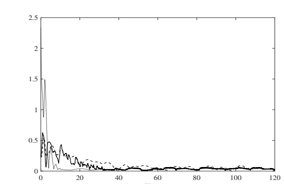
Fig. 1. Effect of the proposed controller (31) on the attitude tracking.



(a) Response of $\|q_e\|$: $\alpha = 0.3, k_1 = k_2 = 0.05$ (solid line), $\alpha = 0.7, k_1 = k_2 = 0.05$ (dashed line), and $\alpha = 0.7, k_1 = k_2 = 0.5$ (dotted line)



(b) Response of $\|\omega_e\|$ (rad/s): $\alpha = 0.3, k_1 = k_2 = 0.05$ (solid line), $\alpha = 0.7, k_1 = k_2 = 0.05$ (dashed line), and $\alpha = 0.7, k_1 = k_2 = 0.5$ (dotted line)



(c) Response of $\|\tau\|$ (Nm): $\alpha = 0.3, k_1 = k_2 = 0.05$ (solid line), $\alpha = 0.7, k_1 = k_2 = 0.05$ (dashed line), and $\alpha = 0.7, k_1 = k_2 = 0.5$ (dotted line)

Fig. 2. Effect of the controller parameters on the performance of the proposed controller.

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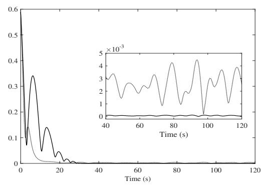
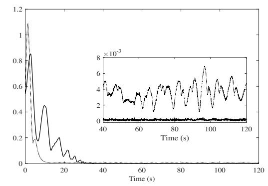
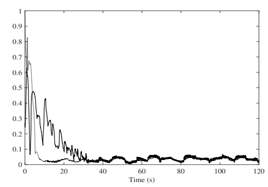


(a) Response of $\|q_e\|$ by the fixed-time controller (solid line) and the finite-time controller (dotted line)

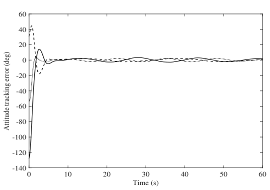


(b) Response of $\|\omega_e\|$ (rad/s) by the fixed-time controller (solid line) and the finite-time controller (dotted line)

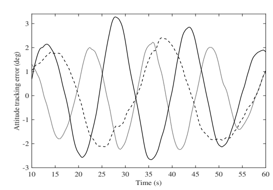


(c) Response of $\|\tau\|$ (Nm) by the fixed-time controller (solid line) and the finite-time controller (dotted line)

Fig. 3. Performance comparison between the fixed-time control law (31) and the finite-time controller [8].

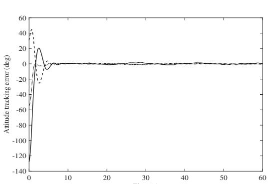


(a) $t \geq 0$ sec

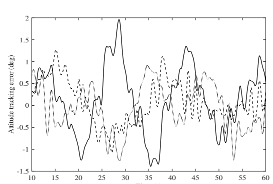


(b) $t \geq 10$ sec

Fig. 4. Attitude tracking error in Euler angles: φ_e (solid line), θ_e (dashed line), and ψ_e (dotted line) (without low-pass filter).



(a) $t \geq 0$ sec



(b) $t \geq 10$ sec

Fig. 5. Attitude tracking error in Euler angles: φ_e (solid line), θ_e (dashed line), and ψ_e (dotted line) (with low-pass filter).

$0.5, \gamma_2 = 0.5, k_1 = 0.05$, and $k_2 = 0.05$. The initial conditions for the observer are $\hat{q}_e(0) = q_e(0)$ and $\hat{v}_e(0) = 0$.

First, the initial conditions are $q(0) = \text{col}(0.07, -0.15, 0.5)$ and $\omega(0) = \text{col}(20, -18, 30)$ deg/s. The measurement noises are considered in the simulation, and the measured attitude is assumed to be $q_m = q + q_n$, where the elements of q_n are generated from a uniform distribution in $n_a[-1, 1]$ with a magnitude of $n_a = 0.00008$. It could be pointed out that a magnitude of 0.00008 on MRPs measurements is physically equal to about 0.0183 degree error in Euler angles with a 3-2-1 rotation sequence. The results are depicted in Fig. 1. It can be observed that the attitude tracking errors in Euler angles (i.e. φ_e, θ_e, and ψ_e) are less than 0.02 degree when $t \geq 50$s. This demonstrates that the proposed control law can provide good attitude performance even in the absence of angular velocity measurements as well as in the presence of uncertainty and noises.

Second, the effect of the parameters (i.e. α, k_1, and k_2) on the performance of the proposed controller is addressed. The initial conditions are considered to be $q(0) = \text{col}(-0.2, -0.5, 0.6)$ and $\omega(0) = \text{col}(-15, 20, -20)$ deg/s. The results are shown in Fig. 2. It is found that the attitude tracking errors become larger as the parameter α increases while better attitude control performance can be achieved if the controller gains increase to $k_1 = k_2 = 0.5$ when $\alpha = 0.7$. However, larger control torques are required. The results indicate that the better the attitude control performance, the smaller the parameter α and the larger the parameters k_1 and k_2 are required.

Third, the performance of the present fixed-time velocity-free controller (31) is compared with the finite-time velocity-free controller in [8]. The controller gains are $k_1 = k_2 = 0.5$ and the values of other parameters are chosen the same as those in [8]. The results are shown in Fig. 3. It can be seen that the finite-time controller can provide faster convergence rate, because the controller gains used in the finite-time controller (i.e. $k_1 = k_2 = 0.5$) are ten times as much as those used in the proposed controller (i.e. $k_1 = k_2 = 0.05$). However, during the steady-state stage, the fixed-time control law (31) can lead to better attitude control performance than the finite-time control law in [8].

Finally, the performance of the proposed controller is examined in a worse case. In this example, the external disturbance is supposed to be $\vartheta = 100\text{col}(\sin(t/10), \cos(t/10), \sin(t/5))$ mNm and the elements of the measurement noise q_n are generated from a Gaussian distribution. Here, the mean and standard deviation of the Gaussian distribution are taken as $\mu = 0$ and $\sigma = 0.01$. Note that a magnitude of $3\sigma = 0.03$ on MRPs measurements is physically equivalent to about 7.25 degree errors in Euler angles with a 3-2-1 rotation sequence. The controller parameters are $k_1 = k_2 = 0.5$ and $\alpha = 0.5$, and the other parameters are chosen as the same as those in the above cases. It can be observed from Fig. 4 that the attitude tracking errors in Euler angles (i.e. φ_e, θ_e, and ψ_e) are less than 3.4 degrees at the steady-state stage when the magnitude of the measurement noise is as large as about 7.25 degrees. This demonstrates that the performance of the proposed controller can be degraded with the increase of the measurement noise's magnitude. To reduce the effect of the measurement noise, like the works in [3], [8], the measured attitude q_m is passed through a low-pass filter $1/(1+0.1s)$, where s is the Laplace variable. Then, the output of the filter, q_f, is applied in the controller and the observer. The results are shown in Fig. 5. It can be found that the use of the low-pass filter can reduce the attitude tracking errors which are less than 2 degrees at the steady-state stage.

Remark 12. The results indicate that the measurement noise may have an important influence on the performance of the

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controller, and the effect of the measurement noise can be reduced by using a low-pass filter. In a recent paper [43], under the assumption of the boundedness of the measurement noise and its first two derivatives, the attitude measurement noise is considered as mismatched disturbance. Then, the adaptive control approach is used to cope with the mismatched disturbance.

V. Conclusions

The problem of velocity-free attitude tracking control for spacecraft has been studied in the paper. By use of the homogeneous method, a novel fixed-time attitude controller was developed. The proposed control scheme can guarantee that the attitude state of the spacecraft converges to a time-varying reference attitude within fixed time. Numerical simulations were conducted to illustrate that the present controller can produce good attitude control performance even in the presence of measurement noises, external disturbances and parametric uncertainties and in the absence of angular velocity measurements. Furthermore, numerical comparison between the designed control law and a velocity-free finite-time attitude control scheme in the literature was examined to show that the proposed fixed-time controller can lead to higher attitude accuracy and stronger robustness against uncertainties than the finite-time control law. One of our future works will extend the control scheme developed here to attitude coordination control for spacecraft formations.

Appendices

Proof of Proposition 1

Let $\alpha \in (0, +\infty)$. Referring to [32], we can verify that $\bar{V}_1(\eta)$ is well defined, positive definite, radially unbounded, of class $C^1(R^6, R)$, and homogeneous of degree 2 w.r.t. the dilation $\delta^r_{1\lambda}(\eta)$. Further, a similar proof for Part (c) can be found in [38]. Hence, only the proof for Parts (b) and (d) is considered.

(b) If $\eta = 0$, we can conclude that $c_1 V_x^2 \le \bar{V}_1(\eta) \le c_2 V_x^2$ holds for some positive constants c_1 and c_2. Let $\eta \in S^6(x)$. Since the set $S^6(x)$ is compact, there exist positive constants c_1 and c_2 such that $c_1 \le \bar{V}_1(\eta) \le c_2$ when $\eta \in S^6(x)$, i.e. $c_1 V_x^2 \le \bar{V}_1(\eta) \le c_2 V_x^2$.

For any $\eta \in R^6 \setminus \{0\}$, by Lemma 1, there exist $\lambda_1 > 0$ and $\lambda_2 > 0$ such that $\eta = \delta^r_{1\lambda_1}(\bar{\eta})$ and $\tilde{\eta} = \delta^r_{1\lambda_2}(\bar{\eta})$, where $\bar{\eta} \in S^6(\eta) = \{\eta \in R^6 |\ \|\eta\| = 1\}$ and $\tilde{\eta} \in S^6(x)$, and it follows that $\eta = \delta^r_{1\lambda_1/\lambda_2}(\tilde{\eta}) = \delta_{1\lambda}(\tilde{\eta})$, where $\lambda = \lambda_1/\lambda_2$. Thus, we have $\bar{V}_1(\eta) = \bar{V}_1\left(\delta^r_{1\lambda}(\tilde{\eta})\right) = \lambda^2 \bar{V}_1(\tilde{\eta})$ and $\lambda^2 c_1 V^2_{x(\tilde{\eta})} \le \lambda^2 \bar{V}_1(\tilde{\eta}) \le \lambda^2 c_2 V^2_{x(\tilde{\eta})}$. Since V_x is homogeneous of degree 1 w.r.t. the dilation $\delta^r_{1\lambda}(\eta)$, we have $c_1 V^2_{x(\eta)} \le \bar{V}_1(\eta) \le c_2 V^2_{x(\eta)}$. Therefore, $c_1 V_x^2 \le \bar{V}_1(\eta) \le c_2 V_x^2$ for all $\eta \in R^6$.

(d) Note that $\frac{\partial \bar{V}_1(\delta^r_{1\lambda}(\eta))}{\partial \delta^r_{1\lambda}(\eta_{1i})} = \lambda \frac{\partial \bar{V}_1(\eta)}{\partial \eta_{1i}} (i = 1, 2, 3)$, which implies that $\frac{\partial \bar{V}_1(\eta)}{\partial \eta_{1i}}$ is homogeneous of degree 1 w.r.t. the dilation $\delta^r_{1\lambda}(\eta)$. Similarly, we can obtain that $\frac{\partial \bar{V}_1(\eta)}{\partial \eta_{2i}}$ is homogeneous of degree $2 - \alpha_1$ w.r.t. the dilation $\delta^r_{1\lambda}(\eta)$. Using the homogeneity property, we can show that $\left|\frac{\partial \bar{V}_1(\eta)}{\partial \eta_{1i}}\right| \le c_{3i} V_x$ and $\left|\frac{\partial \bar{V}_1(\eta)}{\partial \eta_{2i}}\right| \le c_{4i} V_x^{2-\alpha_1}$ for some positive constants c_{3i} and c_{4i} and for all $\eta \in R^6$.

Proof of Corollary 1

The proof is similar to that of Theorem 1, except for the presence of the lumped uncertainty Υ.

Part 1: $V_y > 1$. Considering the Lyapunov function $V_2(\eta) = \bar{V}_2(\eta)/c_5$ and using (16), we obtain $\dot{V}_2(\eta) \le -\varrho_1 V_2^{(5-\alpha)/4} + \frac{1}{c_5\theta}\frac{\partial \bar{V}_2}{\partial \eta_2}\Upsilon$. Since the control torque and external disturbances are assumed to be bounded and $E \in \Omega_{\Delta_1}$, there exists a positive constant Υ_M such that $\|\Upsilon\| \le \Upsilon_M$. Then, we have

$$\begin{aligned}\dot{V}_2(\eta) &\le -\varrho_1 V_2^{(5-\alpha)/4} + \frac{\Upsilon_M}{c_5\theta} V_2^{(2-\beta_1)/2} \\ &\le -\frac{\varrho_1 c_5\theta - \Upsilon_M}{c_5\theta} V_2^{(5-\alpha)/4}. \end{aligned} \tag{32}$$

If we select the parameter θ such that $\varrho_1 c_5 \theta - \Upsilon_M > 0$, then we can conclude that $V_2(\eta)$ converges to $V_2(\eta) \le 1$ (i.e. $V_y \le 1$) within fixed time.

Part 2: $V_y \le 1$. Considering the Lyapunov function $V_1(\eta) = \bar{V}_1(\eta)/c_2$ and using (18), we obtain

$$\begin{aligned}\dot{V}_1(\eta) &\le -\varrho_2 V_1^{(\alpha+3)/4} + \frac{1}{c_2\theta}\frac{\partial \bar{V}_1}{\partial \eta_2}\Upsilon \\ &\le -\varrho_2 V_1^{(\alpha+3)/4} + \frac{\Upsilon_M}{c_2\theta} V_1^{(2-\alpha_1)/2} \\ &= -\varrho_2 V_1^{(2-\alpha_1)/2}\left(V_1^{\alpha/2} - \frac{\Upsilon_M}{\varrho_2 c_2 \theta}\right). \end{aligned} \tag{33}$$

From the above equation, we can conclude that V_1 converges to the region $V_1 \le \left(\Upsilon_M/(\varrho_2 c_2\theta)\right)^{2/\alpha}$ within fixed time, i.e. x converges to the region $\|x\| \le (c_1/c_2)^{1/2}\left(\Upsilon_M/(\varrho_2 c_2\theta)\right)^{1/\alpha}$ within fixed time.

Let $\theta > 1$, then we can obtain

$$\begin{aligned}\|o_e\| &\le \sum_{i=1}^{3}(|\tilde{q}_{ei}| + |\tilde{v}_{ei}|) \le \theta \sum_{i=1}^{3}(|\eta_{1i}| + |\eta_{2i}|) \\ &= \theta \sum_{i=1}^{3}(|x_{1i}| + |x_{2i}|^{\alpha_1}) \le \theta \sum_{i=1}^{3}(|x_{1i}|^{\alpha_1} + |x_{2i}|^{\alpha_1}) \\ &\le 6^{1-\frac{\alpha_1}{2}}\theta \|x\|^{\alpha_1} \le 6^{1-\frac{\alpha_1}{2}}\theta\left(\frac{c_1}{c_2}\right)^{\frac{\alpha_1}{2}}\left(\frac{\Upsilon_M}{\varrho_2 c_2\theta}\right)^{\frac{\alpha_1}{\alpha}} \\ &= 6^{1-\frac{\alpha_1}{2}}\left(\frac{c_1}{c_2}\right)^{\frac{\alpha_1}{2}}\left(\frac{\Upsilon_M}{\varrho_2 c_2 \theta^{\frac{1-\alpha}{2\alpha_1}}}\right)^{\frac{\alpha_1}{\alpha}} \end{aligned} \tag{34}$$

where Lemma 3 and the fact that $\|x\| \le 1$ have been used.

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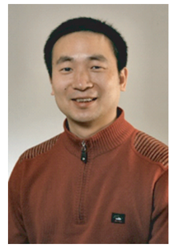

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